MATHEMATICS SPECIAL METHODOLOGY

1. **MODULE CODE**: EM40220 **FACULTY:** EDUCATION
2. **MODULE TITLE**: SPECIAL METHODOLOGY IN MATHEMATICS
3. **LEVEL:** 4 **SEMESTER:** 1 **CREDITS:** 10
4. **FIRST YEAR OF PRESENTATION**:2016
5. **ADMINISTERING FACULTY**: EDUCATION
6. **PRE-REQUISITE:**
7. **ALLOCATION OF STUDY AND TEACHING HOURS**

Activity	**Student hours**
Lectures	30
Seminars/workshops	10
Practical classes/laboratory	6
Structured exercises	4
Set reading etc.	10
Self-directed study	20
Assignments – preparation and writing/marking	12
Examination – revision and attendance/marking	8
Field trip	-
Total	100

8. **BRIEF DESCRIPTION OF AIMS AND CONTENT**

The module aims at giving students a deep knowledge of specific teaching methods of mathematics (whole process of lesson preparation, delivery and evaluation).

9. **LEARNING OUTCOMES**

Knowledge and Understanding

Having successfully completed the module, students should be able to:

(i) Demonstrate a broad knowledge base in the area of teaching specific methods.

Cognitive/Intellectual skills/Application of knowledge

Having successfully completed the module, students should be able to:

(ii) Evaluate evidenced-based solutions to defined teaching problems/issues

Communication / ICT / Numeracy / Analytic Techniques / Practical Skills

Having successfully completed the module, students should be able to:

(iii) Convey complex information to a variety of audiences and for a variety of purposes in relation with teaching.

General transferable skills

Having successfully completed the module, students should be able to:

(iv) Take the lead in preparing scheme of work and lesson plan.
(v) Have relevant skills in teaching mathematics.

9. INDICATIVE CONTENT

Learning/teaching processes and methods, class management and handling, preparation of scheme of work, lesson plan and teaching/learning aids, and evaluation/assessment, micro-teaching.

10. LEARNING AND TEACHING STRATEGY

The theoretical lessons will be followed by tutorials/seminars, guided readings and exercises, as well as student practical works (individual or by group)

11. ASSESSMENT STRATEGY

Assessment will consist of written/oral assignments and tests.

12. ASSESSMENT PATTERN

Component	**Weighting (%)**	**Learning Objectives Covered**
In-Course Assessment		
CATs +Assignments	40	i-v
Final Assessment	**60**	i-iv

13. STRATEGY FOR FEEDBACK AND STUDENT SUPPORT DURING MODULE

After each assessment or assignment, students will get their results and will be advised for better performance.

Chap I: INTRODUCTION TO SCIENCE TEACHING

I.1 INTRODUCTION

A science teacher should not only have an adequate understanding of science but should equally be familiar with the processes of science. The relevance of having an accurate knowledge of both the nature and the relationship of science, technology and society cannot be under estimated in the dynamic world.

I.1.1 Sources of knowledge

Here are philosophical sources of knowledge from untested opinion to highly systematic styles of thinking:

1. **Empiricism** is said "to denote observations and propositions based on sensory experience and / or derived from such experience by methods of inductive logic, including mathematics and statistics'. Empiricists attempt to describe, explain, and make predictions by relying on information gained through observation**.** This book is fundamentally concerned with empiricism – with the design of procedures to collect factual information about hypothesized relationships that can be used to decide if a particular understanding of a problem and its possible solution are collect.

2. **Rationalism:** where reasoning or applying judgement is a primary source of knowledge. Rationalism differs from empiricism in that rationalists believe all knowledge can be deduced from known laws or basic truths of the nature (for example, gravity). This is claimed to be possible because underlying laws structure the world logically. From the time of Sir Francis Bacon to the present, adherents of rationalism have maintained that problems are best understood and resolved through formal logic or mathematics. Such efforts, of course, operate independently of observation and data collection.
3. **Untested opinion:** people cling to untested opinion despite contrary evidence. In indoctrination programs of less enlightened organizations, it is not unusual for new employees to hear "that's the way we've always done it here'- a remark that confuses entrenchment and habit with efficiency. Managers will find little to improve their understanding of reality from untested opinion even though human nature indicates they should be prepared to cope with its use by contemporaries when searching for solutions to management dilemmas.
4. **Realism:** do objects exist independently of our knowledge of their existence? The essence of realism is that what the senses show us as reality is the truth: that objects have an existence independent of the human mind. The philosophy of realism is that there is a reality quite independent of the mind. In this sense, realism is opposed to idealism, *the theory that only the mind and its contents exist*. Realism is a branch of epistemology which is similar to positivism in that it assumes a scientific approach to the development of knowledge. This meaning becomes clearer when two forms of realism are contrasted.

- *Direct realism*

It says that what you see is what you get: what we experience through our sense portrays the world accurately.

- *Critical realism*

Critical realists argue that what we experience are sensations, the images of the things in the real world, not the things directly. Critical realists point out how often our senses deceive us. ….

A simple way to think about the difference between direct and critical realism is as follows. Critical realism claims that there are two steps to experiencing the world. First, there is the thing itself and the sensations it conveys. Second, there is the mental processing that goes on sometime after that sensation meets our senses. Direct realism says that the first step is enough.

The critical realist, on the other hand, would recognize the importance of multi-level study (e.g at the level of the individual, the group and the organisation). Each of these levels has the capacity to change the researcher's understanding of which is being studied. This would be the consequence of the existence of a greater variety of structures, procedures and processes and the capacity that these structures, procedures and processes have to interact with one another. We, therefore, would argue that the critical realist's position that the social world is constantly changing is much more in line with the purpose of business and management research which is too often to understand the reason for phenomena as precursor to recommending change.

5. **Ontology**: what assumptions do we make about the way in which the world works?

Ontology is concerned with nature of the reality.

- **Objectivism**

It portrays the position that the social entities exist in reality external to social actors concerned with their existence (to adopt an objectivist stance to the study of a particular aspect…).

- **Subjectivism**: understanding the meanings that individuals attach to social phenomenon

The subjectivist view is that social phenomenon is created from the perceptions and consequent actions of social actors. What is more, this is a continual process in that through the process of social interaction these social phenomena are in constant state of revision.

6. **PERENNIALISM**

- it focuses on the education of rational aspect of people; They can use their reason to control their appetites and passions;
- Human nature is universally consistent: therefore education should be the same for everyone;
- Knowledge is universally consistent: therefore there are certain basic subject matter that should be taught to all people
- The subject matter, not the student should stand at the center of the educational endeavor

-The focal point is activities designed to discipline the mind by giving difficult mental exercises.ie drill, rote memory, logic and computation

-Develop the will power that will be needed later in life when facing difficult tasks

- The great works of the past are a repository of knowledge and wisdom.

7. **HUMANISM**

- Learning environments should make students free from intense competatition, harsh discipline and fear of failure;
- Standardized testing and mass teaching is discouraged;

- Discouraged formal classroom and instead encouraged open classroom;
- Schools should seek to develop independent and courageous people who are able to deal with the changing complexities of the modern world;
- It advocates for open classroom, free school and schools without failure

8. **Behaviorism**

- Human beings are highly developed animals who learn in the same way that animals learn;
- Techniques of teaching can be refined through experimentation with animals;
- Education is a process of behavioral engineering;
- People are programmed to act in certain ways by their environment. Therefore behavior can be modified by manipulating environmental reinforces;
- The task of education is to create learning environments**.**

9. **EXISTENTIALIM**

- The individual is the center of education;
- The teacher is one who helps students to find possible answers;
- No two students are alike;
- Deferent education for different students;
- Students should have freedom in the selection of subject matter;
- Curriculum should be flexible;
- It emphasizes on humanities because it deals with meaningful aspects of life
- Curriculum should have many options for students who desire to learn

I.1.2 The thought process: Reasoning

- **Deduction** is a form on inference that purports to be conclusive – the conclusions necessarily follow from the reasons given. These reasons are said to *imply the conclusion* and *represent a proof*. This is a much stronger and different bond between reasons and conclusions than is found with induction. For a deduction to be correct, it must be both **true** and **valid**.
  - ✓ Premises (reasons) given for the conclusion must agree with the real world (**TRUE**)
  - ✓ The conclusions must necessarily follow from the premises (**VALID**)

A deduction is valid if it is impossible for the conclusion for to be false if the premises are true. Conclusions are not logically justified if one or more premises are untrue or argument form is invalid.

- **Induction:** inductive argument is radically different. There is no such strength of relationship between reasons and conclusions in induction. To induce is to draw a conclusion from one or more particular facts or pieces of evidence. The conclusion explains the facts, and the facts support the conclusion.
- **Combining induction and deduction:** induction and deduction are used in reasoning in a sequential manner. Johnson Dewey describes this process as "**the double movement of reflective thought"**. Induction occurs when we observe a fact and ask "**why is this**"?. *In answer to this question, we advance a tentative explanation* (**hypothesis**). The hypothesis is plausible if it explains the event or condition (fact) that prompted the question. *Deduction is **the process by which we test whether the hypothesis is capable of explaining the fact**.*

I.1.3 Reflective thought and the scientific method

Induction, deduction, observation and hypothesis can be combined in a systematic way to illustrate the scientific method. The ideas that follow, originally suggested by Dewey and other problem solving analysis, represent one approach to assessing the validity of conclusions about observable events. They are particularly appropriate for researchers whose conclusions depend on empirical data.

1. Encounters a curiosity, doubt, barrier, suspicion, or obstacle;
2. Struggles to state the problem: asks questions, contemplates existing knowledge, gathers facts, and moves from an emotional to an intellectual confrontation with the problem;
3. Proposes hypotheses to explain the facts that are believed to be logically related to the problem;
4. Deduces outcomes or consequences of the hypotheses: attempts to discover what happens if the results are in the opposite direction of that predicted or if the results support the expectations;
5. Formulates several rival hypotheses;
6. Devises and conducts a crucial empirical test with various possible outcomes, each of which selectively excludes one or more hypotheses;
7. Draws a conclusion, an inductive inference, based on acceptance or rejection of the hypotheses;
8. Feeds information into the original problem, modifying it according to the strength of the evidence.

Eminent scientists, who claim there is no such thing as the scientific method, or if it exists, it is not revealed by what they write, caution researchers about using template-like approach.

I.2 THE NATURE OF SCIENCE

There are many ways that we can define the term science.

Mohan (1995) defined science as: *cumulative and endless series of empirical observations which result in the formation of concepts and theories, with both concepts and theories being subject to* *modification in the light of further empirical observations".* Mohan explained that science is an interconnected series of concepts and conceptual schemes that have developed as a result of experimentation and observation and fruitful for experimentation and observation.

An ordinary man in the street who sees sciences through its applications views science as a body of scientific information.

A scientist, who may have vast knowledge about scientific ideas, concepts and principles, sees science as a method by which hypothesis are tested.

On other hand, philosopher may look at science as a system of questioning the reality or truthfulness of facts, concepts or knowledge. Otherwise science could be defined as a consisting of three main parts:

- A body of knowledge;
- A methods of inquiry, a system of investigating;
- An attitude towards life, a way of thinking.

I.2.1 Science as a body of knowledge

Science is considered as a body of knowledge because this knowledge which includes facts, concepts, theories, principles and laws, has been tested and confirmed by scientists. One characteristic about this body of knowledge is that it is subject to error and change. An individual that is scientifically literate should be able to differentiate between that is confirmed and supported by evidence and that knowledge which is merely speculative, without supporting the evidence. As a body of knowledge science consist of:

- Experimentally verified principles, laws and theories;
- A mathematical component that expresses relationships in the economic language of mathematics as a basis for prediction;
- A set of hypotheses about how things might be related in nature;
- A theoretical component that explains natural phenomena using unifying theories.

I.2.2 Science as a method of inquiry (World problems)

Science and scientists educators believe that there should be some degree of flexibility while specifying the number of steps or stages to be involved in scientific method. This is because scientific activities are not the same to all practitioner of the discipline. However, certain steps are believed to constitute a scientific method. According to Abdullahi (1981) and Mohan (1995) there are six main steps called the Hypothetical-deductive method.

- The identification of the problem;
- The collection of relevant information;
- Statement of a hypothesis based on observation gathered;

- The making of deductions from the hypothesis;
- The hypothesis is tested through observations;
- Depending on the outcome of the experiment, the working hypothesis is accepted, modified or rejected.

These steps serve as guidelines, while a true scientist deals with the problem with inspiration, imagination and creativity. Discipline would however help the scientist to reduce subjectivity in his/her activities.

Some scientists use the method of induction while others use the methods of deduction. In induction, they reason from particular instances to arrive at a generalization; while in deduction they apply generalization to particular instances. Great scientists are known to use creativity and aesthetic values in making important discoveries.

Some scientific activity is focussed on verifying knowledge claims, while some scientists prefer to falsify previous knowledge claims. The key process-skills of science include: observing, classifying, measuring, recording, hypothesizing, designing, interpreting, extrapolating, discussing, explaining, applying, evaluating, synthesizing, analysing, inferring, interpolating and communicating. The process of scientific enquiry has three distinct phases:

- **Exploration:** that involves finding out about relationships in nature;
- **Elaboration:** that involves the assigning of meaning to patterns of findings through explanation;
- **Application:** that involves the use of new knowledge to deal with new situations.

I.2.3 Science as an attitude toward life

An attitude could be defined as a condition of mind that brings in imagination and emotional states which are the outcome of previous experiences that inclines someone towards or away from something. It could be described as a condition of readiness for a certain type of activity. Attitude direct behaviour, and are therefore important in learning. A scientific attitude can be developed through the learning of science. An individual with a scientific attitude can be described as:

- Critical in observation and thought;
- Open minded;
- Respectful of others' point of view and ready to change his/her decision on presentation of new and convincing evidence;
- Curious to know more about things around him/her, wants to know "why" and "how" of things he/she observes;
- Objective in his/her approach to problems;

- Does not believe in superstitions;
- Truthful in his/her observations and draws conclusions based on accurate facts;
- Unbiased and impartial in his/her judgements;
- Adopts a planned procedure in solving a problem;
- Believes that truth never changes but his/her ideas on what is true may change as he/she gains better understanding of that truth;
- Accepts no conclusions as final or ultimate;
- Seeks to adopt different techniques and procedures to solve problems;
- Selects the most recent, authoritative and accurate evidence related to problems;
- Seeks the facts and avoids exaggeration.

I.2.4 Dynamic nature of science

While the idea that science as a body of accumulated knowledge is true, science is however dynamic in nature. Scientific information is constantly being rearranged and reoriented in the light of latest developments due to knowledge explosion. As teachers, it is our responsibility to explain the dynamic nature of science to students/learners so as to enable them to utilise the knowledge in their daily life. Such knowledge will help the students to develop positive attitudes towards science as a discipline.

According to Carl Popper, science is asset of conjectures and refutations. This makes scientific knowledge tentative. Imre Lakatos describes science as a set of competing research programmes, while Thomas Kuhn explains that the progress of science in driven by paradigm change in revolutionary ideas. These assertions show that science is dynamic in nature, and this image of science should be integrated into science education.

I.3 AIMS, OBJECTIVES, PROBLEMS AND ISSUES IN TEACHING SCIENCES

I.3.1 Aims and objectives of teaching science in schools

Science and society in general are dynamic in nature. The exposition of knowledge and the consequents changes in curricula and methods of instructions have created new goals for science teaching.

Objectives of science teaching or any other subject are now determined by relevancy to the learner, society, school and other variables within the environment. The general objectives of science teaching in the school curriculum include the followings:

- Better understanding of the nature of science;
- The acquisition of scientific skills;
- The development of scientific attitudes;

- The training of learners in scientific methods;
- To develop in learners, interest and appreciation of science;
- To assist and to integrate the learner fit himself/herself into society;
- To assist the learner acquires a suitable career based on his/her interest.

I.3.2 Rwandan national policy on science teaching and ICT in education

The Rwandan ministry of Education (2003) has adopted the following national policy on science teaching and ICT education:

Policy: Science and Mathematics teaching and ICT shall be at the heart of all levels of education. Links will be started between all institutions of learning from primary to higher education.

Strategies: Accordingly, the following activities shall be carried out:

- Training a critical mass of science and ICT teachers;
- Ensuring practical skills and providing science equipment and computers to identified schools and progressively to all schools as means allow;
- Establishing model centres of excellence in science, mathematics and ICT at secondary level;
- Developing the ICT curriculum for all levels of education;
- Ensuring that science and mathematics programmes at primary and secondary levels are coherent;
- Establishing partnerships between educational institutions of different levels.

The four main purposes for teaching science would be:

- Preparing learners to become future scientists;
- Preparing learners for appropriate career opportunities in science related fields;
- Providing a bridge to technological advancement;
- Producing citizens with adequate scientific literacy.

The science teachers will therefore require attention to these policy aspects in planning for both teaching and evaluation of learning programs

I.3.3 Problems and issues in science teaching

Based on research, a number of problems in science teaching have been identified. They include the followings:

- Lack of human resources;
- Inadequate funding;
- Lack of sufficient equipment, materials and laboratories;
- Bookish and examination oriented curriculum;
- Memorization of lecture notes-popularly known as rote learning;
- Large population of students in a classroom;
- Hunger (due to lack of feeding) while in the classroom;
- Poverty related problems;
- Traumatized students;
- Poor teaching methodology.

The problem of lack of resources and inadequate funding can be addressed through:

- Borrowing resources from other institutions like libraries, resource centres and neighbouring schools;
- Creative improvisation from locally available materials;
- Cost sharing among stakeholders;
- Use income generating activities;
- Soliciting of grants and donations from appropriate sources;
- A deliberate National effort to manufacture science kits.

Other challenges may be addressed through

- Proper orientation of science students;
- Use of motivational techniques;
- Use of guidance and counselling strategies;
- Good training of teachers in methodology.

I.3 THEORIES OF LEARNING AND THEIR IMPLICATIONS FOR SCIENCE TEACHING

I.3.1 Application of Ausubel's theory to science teaching

The main points in Ausubels' theory are:

- The most important basis for learning is what the learner already knows;
- The use of advance organizers promotes meaningful learning.

The application of this theory to science teaching is that dissemination of knowledge should be approached structurally. By the implication, science teachers should begin instructional presentation with a set of organised statements. These statements, which may be called subsumes, should be broad/general and inclusive for effective linking and bridging for the body of knowledge that is already held by the students and the new learning material or concepts, principles which are to be learned or introduced.

I.3.2 Application of Gagne's theory to science teaching

Gagne believes that new knowledge is easily acquired if it is well sequenced and arranged in a hierarchical manner. The theory implies that concepts, principals or knowledge or materials meant for learning should be sequentially structured by the teacher. If we are to consider a lesson plan based on Gagne's theory- a teacher could begin by asking such questions as: at the end of the lesson or instruction what body of knowledge, concept, idea or skill do I want the learner possess? It should be noted that at this juncture, the science teacher has started the process of task analysis to identify the prerequisites for the learning hierarchy or simply a hierarchy of task.

N.B: Like Ausubel, Gagne sees the achievement of meaningful learning through the movement from a simple to a complex task or from the known to unknown or from concrete/easy to abstract.

I.3.3 Bruner's theory of learning by discovery and science teaching

Bruner's learning theory emphasises the acquisition of the knowledge, concepts, facts, theories or laws through a discovering learning. Discovery in the case is referred to as methods of obtaining ideas, factors, concepts, for oneself by use of one's mental processes (i.e: analysing, investigating, manipulating, controlling, inferring or concluding).

According to Bruner, there are two types of discovering processes:

1. A student spontaneously recognises a new situation that is familiar to one of the elements in his existing structure of knowledge and he/she easily assimilates it;
2. A new structure is incompatible to the existing structure of knowledge. The learner first restructures his/her cognitive framework so that the new learning materials can be accommodated. By implication, assimilation and accommodation are two forms of discovery learning (Abdulallahi, 1982).

Bruner also emphasizes that learners can learn any piece of new knowledge so longas the knowledge is packaged into small manageable units, one at a time. The implication of Bruner's theory for science teaching is that science teachers should:

- Package new ideas in small units that can be gradually developed;
- Deliberately create problems for the students by introducing some potential incongruities or contradictions among sources of information, which are given in

the process of information. In line with this theory such inconsistencies lead to what can be described as “intellectual discomfort” that will later stimulate the learner to initiate individual discoveries through cognitive restructuring.

I.3.4 Piaget’s cognitive theory and science teaching

One of the most important principles Piaget’s theories of developmental psychology is that the selection of content for different levels of learning should take into account the development of learner abilities from the pre –operational, through concrete operational to the highest abstract level of thinking. The theory has become an important guide in the selection and arrangement of science materials for both primary and junior secondary school pupils.

I.3.5 Constructivism and metacognition in science teaching

Jean Piaget while seeking an answer to the perennial philosophical question ”How do we come to know what we know?”. He concluded that knowledge cannot be transmitted intact from one person (e.g: teacher) to another (e.g: student) people must construct their own knowledge and their own understandings. Learning does not occur by transmitting information from the teacher or the textbook (or the video or the demonstration) to the child’s brain. Instead, each child constructs his/her own meaning by combining earlier information (prior knowledge), information with the new knowledge provides personal meaning to the learner or student at the end.

It should therefore be noted that believe, that people build their own knowledge and their own representations knowledge from their own experience and thought is called constructivism. The constructivist theory of learning considers learning as a process in which students actively construct their knowledge of the situation at hand based on the already existing conception or misconception.

I.3.6 Metacognitive and assessment

We start with discussion, ask some questions, listen to the answers, and talk with the students. Based on this discourse, they can quickly estimate the depth of students’ prior knowledge. This type of informal assessment can be used to calibrate the instruction to help students gain both contents knowledge (whether it be factual, conceptual, or procedural) and metacognitive knowledge.

It should be realised that from the informal assessment conversations, teacher also may be able to make inferences about the level of metacognitive knowledge of individual students. This information about individual students can be used to adapt instruction to individual differences; there is variance in the content knowledge that students bring to classroom of 20-30 students. Teacher can talk to students individually or to small group of students to estimate levels of metacognitive knowledge. Finally, more formal questionnaires and interview procedures can be used to assess students’ metacognitive knowledge concerning their learning strategies as well as their knowledge about different tasks and context.

I.3.7 Misconception in science

The term **misconception** is sometimes called **preconceived notions or non-scientific beliefs; native theory; mixed conceptions’ or conceptual misunderstanding.** Unfortunately, both teachers and students who hold misconception are not aware that their ideas are incorrect. When they are simply told that they are wrong, they often have a hard time giving up their belief, especially if they have had such a belief for a long time.

Driver an Easley (1987) and Hansein (2005) contend that misconceptions serve the needs of the person who hold them and that erroneous ideas may come from strong word association, confusion, conceptions, conflicts, or lack of knowledge. According to them, some alternative conception judged to be erroneous ideas or misconceptions have these characteristics in common

- They are at variance with conceptions held by experts in the field;
- A single misconception, or a small number of misconceptions, tends to pervasive (shared by many individuals)
- Many misconceptions are highly resistant to change or alteration, at least by traditional teaching methods;
- Misconceptions sometimes involve alternative belief systems comprised of logically linked sets of propositional that are used by students in systematic ways.

Some misconceptions have historical precedence, that is, some erroneous ideas put forward by the students today mirror ideas espoused by early leaders in the field.

I.3.8 Strategies of addressing students' misconception

Blosser (1987) suggested that in teacher education programs, pre-service teachers should be assisted so as to develop ideas about conceptual change in learning. Teacher-educators must realize that their students have conception about teaching and learning that are different from those the teacher-educators hold. Therefore, teacher-educators and trainers should work towards changing students' misconceptions. A number of studies admit that the following procedures are necessary while addressing students' misconceptions:

- Start with students' ideas and devise teaching strategies to take some account of them;
- Provide more structured opportunities for students to talk through ideas at length, both in small groups and whole class discussions;
- Begin with known and familiar examples;
- Introduce some science topics into the curriculum at earlier grade levels, drawing on out-of-school knowledge.

I.3.9 Learner centred instruction in science

The phrase "Learner centred instructions" or approaches is described by Eggen and Kauchak (2001) to include instructions which learners, with the teachers' guidance, are made responsible for constructing their own understanding, meaning the students needs experience by experimenting on their own in order to understand.

Aggarwal (2005) in his review described the approach as self activity where the student is at the centre of concern, whether explicitly and implicitly. Learner-centred specialists tend towards the fact that science education is a process of leading out rather imparting knowledge.

The followings are the need for learner-centred instruction and its implication in science teaching

- The learner is the agent of his own learning, because out of the three major components of the learning situation: the learners, the teacher and the environment, then pride of place is given to the learners. The implication of this is that science curriculum must be thought of in terms of activities and experiences which appeal most to the learner;
- Student learn best when they are actively involved. Based on this provision should be made in such a way that learning take place through continuous process of interaction between the students and institutional materials/environment;
- It should be realized that the learner-centred approach is more psychological than logical. It emphasizes the process rather than the product;
- The system gives freedom to the learner under the creative and sympathetic guidance of the teacher;
- Note that one single exposure to an experience, in science learning, does not affect all the necessary coordination of the physical and mental faculties of the student to preserve the net value of the exposure. It means there has to be repetitive and drill to give a certain knowledge and efficiency and tenacity of a skill and value;
- Experts in science education are of the view that the learner's sense of wonder and astonishment and the natural curiosity lead to a learning process which should be encouraged by teachers;

The characteristics of learner-centred approach said that science teachers while in the classroom should take into consideration the prior knowledge/ background, the cognitive and affective thoughts of the students, their development, as well as their social environment, in the line with Bloom's Taxonomy of educational objectives. The followings are characteristics of learner-centred instruction:

- Learners at the centre of the learning process;
- Teacher guides students' construction to understand;
- Teaching for understanding.

Chap II: MATHEMATICS TEACHING AND LEARNING

II.I Introduction

Mathematics teaching today primarily takes place within a professional framework. However, teaching mathematics is a complex and demanding process. Even though being professional is a condition for its success, it is not sufficient. The complexity is successfully resolved by relating math to other sciences. That way we get a process which has to take place harmoniously within several frameworks. The main frameworks are language frameworks, professional frameworks, methodology frameworks, scientific frameworks, pedagogical frameworks and psychological frameworks. As it is not easy to achieve harmony, occasional slips and weaknesses occur in math teaching which significantly influence the quality of math education. That reflects negatively on the aims of modern mathematics teaching which emphasizes involvement of students in independent and research work, developing skills for problem solving and the development of creative thinking and creative skills. Modern mathematics teaching methodology offers various possibilities for solving the above mentioned problem. A teacher can find many possibilities within the scientific frameworks. The foundation of scientific frameworks is the science principle and scientific research methods.

Mathematics is widely regarded as one of the most important subjects; teachers are expected to provide opportunities and the support necessary for all students to learn significant mathematics with depth understanding. Teaching mathematics at different levels is about depth of knowledge rather than the accumulation of isolated facts and procedures.

The philosophy of mathematics describes the nature of mathematics, so understanding the nature of mathematics should able you to make desirable decisions on how to manage a learning environment. The shortest overview of some mathematical theory has four steps:

- Stating basic concepts;
- Axiom formulation;
- Introduction of new concepts;
- Deriving and proving a theorem.

In other words, some scientific math area is a formation of axioms, basic concepts, derived concepts and theorems. In the teaching process, a mathematics teacher helps students to discover and learn new mathematical truths. That knowledge can be obtained in various ways and the bases of all those methods are also concepts and theorems

2.1.1 What is Mathematics?

Mathematics is a language used by scientist, it is an essential element of communication and it has powerful influence In daily life decision. The followings are some answers on the question.

- Mathematics is a tool used by mathematicians and scientists, and also by everyone in the course of daily life;
- Mathematics is a language that uses carefully defined terms, symbols, expressions /arguments and proofs;
- Mathematics is a way of thinking that provides strategies for organizing, analyzing and synthesizing information;
- Mathematics is the study of patterns and relationships.

The nature of mathematics is defined from two prospective: *dynamic prospective and static prospective*

2.1.2 Objectives of mathematics education

The following objectives are specific for mathematics education

- Mathematics contributes to personal and cultural development of an individual. It helps learner to work in a systematic way, to develop clear, logics, coherent and creative reasoning. Learner develops imagination, initiative and flexibility of mind in Mathematics.
- Mathematics is an essential element of communication. It can describe, explain, interpret, present and analyze information;
- Mathematics is a powerful tool. It has utility and practical value. It provides mathematical models and these can enable one to contribute to the resolution of problem encountered in daily life;
- It is useful for learning and comprehension of other subjects like physics, chemistry, medical science, engineering, computer science, etc
- Learner develops confidence in their mathematical abilities and appreciates relationships within mathematics.

2.1.3 Problems and issues in teaching mathematics

The following are problems that are encountered in the teaching of mathematics

- Lack of qualified teachers / many teachers complain for heavy workload;
- Negative attitudes towards mathematics both for teachers and students;
- Lack of necessary prerequisites in some pupils, progression from one concept to the next is not articulated. It is discontinuous especially between primary and secondary levels;
- Lack of appropriated teaching aids. With effort concretization of mathematical concepts is possible;
- Misconceptions: Knowledge representation allows putting into evidence the plurality of possible points of view on a same concept. This appears to be contradictory in one person and would be observed in different situations.

II.2 MATHEMATICS LEARNING AND TEACHING

2.2.1 PIAGET’S THEORY

Piaget's theory of cognitive development is a comprehensive theory about the nature and development of human intelligence. Piaget believed that one's childhood plays a vital and active role in a person's development.

- Piaget's idea is primarily known as a developmental stage theory. The theory deals with the nature of knowledge itself and how humans gradually come to acquire, construct, and use it.
- To Piaget, cognitive development was a progressive reorganization of mental processes resulting from biological maturation and environmental experience. He believed that children construct an understanding of the world around them, experience discrepancies between what they already know and what they discover in their environment, and then adjust their ideas accordingly.
- Moreover, Piaget claimed that cognitive development is at the center of the human organism, and language is contingent on knowledge and understanding acquired through cognitive development.
- Piaget's earlier work received the greatest attention. Many parents have been encouraged to provide a rich, supportive environment for their child's natural propensity to grow and learn. Child-centered classrooms and "open education" are direct applications of Piaget's views.
- Despite its huge success, Piaget's theory has some limitations that Piaget recognized himself: for example, the theory supports sharp stages rather than continuous development (decalage)

2.2.2 The stages of cognitive development that Piaget distinguished are four

Sensorimotor (0-2 years of age): children begin to use imitation, memory and thought. They begin to recognize that objects do not cease to exist when they are hidden from view. They move from reflex actions to goal-directed activity.

Preoperational (2-7 years): Children gradually develop language and the ability to think in symbolic form. They are able to think operations through logically in one direction and they have difficulty seeing another person's point of view.

Concrete operational (7-11 years): Children are able to solve concrete (hands-on) problems in logical fashion. They understand the laws of conservation and are able to classify and seriate. They also understand reversibility.

Formal operational (11-15 years of age): Children are able to solve abstract problems in logical fashion. Their thinking becomes more scientific, they develop concerns about social issues and about identity.

2.2.3 Methods of reasoning

Piaget suggested that when children do not understand or have difficulty with a certain concept, it is due to a too-rapid passage from the qualitative structure of the problem (by simple logical reasoning -**e.g.** a ball existing physically) to the quantitative or mathematical formulation (in the sense of differences, similarity, weight, number, etc).

Conditions that can help the child in his search for understanding according to Piaget are the use of active methods that permit the child to explore spontaneously and require that "new truths" be learned, rediscovered or at least reconstructed by the student not simply told to him (Piaget, 1968). He pointed out that the role of the teacher is that of facilitator and organizer who creates situations and activities that present a problem to the student. The teacher must also provide counterexamples that lead children to reflect on and reconsider hasty solutions. Piaget argued that a student who achieves certain knowledge through free investigation and spontaneous effort will later be able to retain it. He will have acquired a methodology that serves him for the rest of his life and will stimulate his curiosity without risk of exhausting it.

2.2.4 Mathematical understanding

Cognitive scientists and mathematics educators who favour the cognitive science approach have moved well beyond Piaget in describing the way the mind operates. There has been a shift from an organic language of Piaget to a language "highly colored" of computers, (Noddings 1990b) with words such as networks, connections, paths, frame, etc.

Hiebert and Carpenter (1992) propose a framework for considering understanding from the constructivist perspective which would shed light on analyzing a "range of issues related to understanding mathematics." They make a distinction between the external and internal representation of mathematical ideas, pointing out that, to think and communicate mathematical ideas, people need to represent them in some way. Communication requires that the representations be external, taking the form of spoken language, written symbols, drawings or concrete objects. Mathematical ideas become tangible when people can express them. By learning to express their ideas to one another, students can begin to appreciate the nuance of meaning that natural language often masks, but that the precise language of mathematics attempts to distinguish (Lo, Wheatley, & Smith, 1994; Silver, Kilpatrick & Schlesinger, 1990)

Understanding mathematics can be defined as the ability to represent a mathematical idea in multiple ways and to make connections among different representations. In order to think about mathematical ideas these need to be represented internally but these mental representations are not observable. This has led cognitive science to consider mental representations as a field of study (Ashcraft, 1982; Greeno, 1991; Hiebert & Carpenter, 1992).

Connections between external representations of mathematical ideas can be constructed by the learner (Hiebert & Carpenter, 1992) between different fortes of the same idea or between related mathematical ideas. These connections are often based on relationships of similarity or of differences. Connections within the same representation are formed by detecting patterns and regularities.

The relationship between internal representations of ideas constructs a network of knowledge. Understanding then is the way information is represented, so that a mathematical idea, procedure or fact is understood if it is part of an internal network. Networks of mental representations are developed gradually as new information is connected to the network or new ties are constructed between previously disconnected information. Understanding grows as the networks become larger and more organized and can be limited if connections are weak or do not exist becoming useless. (Hiebert & Carpenter, 1992).

Children from a very young age are sensitive to quantity. They perceive differences in number; they see correlation among different numbers of events; their actions contain quantity and they use words referring to basic mathematical events (Gelman, 1980; Ginsburg, 1989).

Various studies (Ginsburg & Baron, 1993; Starkey & Cooper, 1980; Van de Walle & Watkins, 1993) have pointed out the importance of taking into account children's informal mathematical connections as building block for formal mathematics.

Ginsburg (1989) suggests that students need to learn that it is acceptable, "even desirable", for them to connect conventional arithmetic with their own informal knowledge, intuition and invented procedures.

In a study by Carpenter, Ansell, Franke, Fennema and Weisbeck (1993), the results suggest that children can solve a wide range of problems, including problems involving multiplication and division, much earlier than is generally presumed. With only a few exceptions, children's strategies could be characterized as representing or modeling the action or relationships described in the problem. These researchers conclude that young children's problem-solving abilities have been seriously underestimated. They suggest that if from an early age children are motivated to approach problem solving as an effort to make sense out of problem situations, they may come to believe that learning and doing mathematics involves solving problems in a way that always makes sense.

2.2.5 The use of manipulatives in mathematics

For years mathematics educators have advocated using a variety of forms to represent mathematical ideas for students. Physical three-dimensional objects are often suggested as especially useful. Despite the intuitive appeal of using materials, investigations of the effectiveness of the use of concrete materials have yielded mixed results (Bednarz & Janvier, 1988; Bughardt, 1992; Evans, 1991; Hestad, 1991; Hiebert, Wearne, & Taber, 1991; Simon, 1991; Thompson,J., 1992).

P. Thompson (1994) suggests that the apparent contradictions in studies using manipulatives are probably due to aspects of instruction and students' engagement to which the studies did not attend. Evidently, just using concrete materials is not enough to guarantee success according to Baroody (1989). The total instructional environment must be looked into to understand the effective use of concrete materials. In a project by Wesson (1992) for grades 1 and 2, which emphasized exploratory activities with manipulatives, the results suggested that while a much wider range of content than in standard books or tests was covered, there was no loss of arithmetic skills

Children understand when using concrete materials if the materials are presented in a way that helps them connect with existing networks or construct relationships that prompt a reorganization of networks. It is important to consider then, the internal networks that students already carry with them and the classroom activities that promote construction of relationships between internal representations (Hiebert et al, 1991). Manipulatives then can play a role in students' construction of meaningful ideas. Clements and McMillan (1996) and others suggest they should be used before formal instruction, such as teaching algorithms. Clements and McMillan propose that concrete knowledge can be of two types:

Sensory-concrete: it is demonstrated when students use sensory materials to make sense of an idea;

Integrated concrete: it is built through learning. Integrated concrete thinking derives its strength from the combination of many separate ideas in an interconnected structure of knowledge. When children have this type of interconnected knowledge, the physical objects, the actions they perform on the objects and the abstractions they make are all interrelated in a strong mental structure

Ross and Kurtz (1993) offers the following suggestions when planning a lesson involving the use of manipulatives. He suggests that the mathematics teacher should be certain that:

- Manipulatives have been chosen to support the lesson's objectives;
- Significant plans have been made to orient students to the manipulatives and corresponding classroom procedures;
- The lesson involves the active participation of each student;
- The lesson plan includes procedures for evaluation that reflect an emphasis on the development of reasoning skills.

2.2.6 Invented strategies and number sense

In the last few years there have been studies about the idea of students' constructing their own mathematical knowledge rather than receiving it in finished form from the teacher or a textbook (Carpenter, Ansell, Franke, Fennema, Weisbeck, 1993; Markovits & Sowder, 1994). A crucial aspect of students' constructive processes is their inventiveness (Piaget, 1973). Children continually invent ways of dealing with the world. Many of the errors they make can be interpreted as a result of inventions (Ginsburg & Baron, 1993; Peterson, 1991). Similarly, in school mathematics, students rely many times on invented strategies to solve a variety of problems (Carpenter, Hiebert, & Moser, 1981; Carraher & Schliemann, 1985; Ginsburg, 1989). Kamii and Lewis (1993) and Madell (1985) have reported successful work in programs where children are not taught algorithms, but are encouraged to invent their own procedures for the basic operations.

Sowder and Schappelle (1994) suggest that there are common elements found in classrooms that help children acquire good number sense:

1. Sense-making is emphasized in all aspects of mathematical learning and instruction;
2. The classroom climate is conducive to sense making. open discussions about mathematics occurs both in small groups and with the whole class;

3. Mathematics is viewed as the shared learning of an intellectual practice. This is more than simply the acquisition of skills and information. Children learn how to make and defend mathematical conjectures, how to reason mathematically and what it means to solve a problem.

II.3 Principles of Mathematics learning and teaching

2.3.1 The Dynamic Principle

The dynamic principle suggests that true understanding of a new concept is an evolutionary process involving the learner in three temporally ordered stages. The first stage is the preliminary or play stage. The learner here experiences the concept in a relatively unstructured but not random manner. For example, when children are exposed to a new type of manipulative material, they characteristically "play" with their newfound "toy." Dienes suggests that such informal activity is a natural and important part of the learning process and should therefore be provided by the classroom teacher. Following the informal exposure afforded by the play stage, more structured activities are then appropriate. This is the second stage. It is here that the child is given experiences that are structurally similar (isomorphic) to the concepts to be learned. The third stage is characterized by the emergence of the mathematical concept with ample provision for reapplication to the real world.

The completion of this cycle is necessary before any new mathematical concept can become operational for the learner. Dienes referred to the process as a learning cycle (Dienes & Golding, 1971). The dynamic principle establishes a general framework within which learning of mathematics can occur. The remaining components should be considered as existing within this framework.

2.3.2 The Perceptual Variability Principle

The perceptual variability principle suggests that conceptual learning is maximized when children are exposed to a concept through a variety of physical contexts or embodiments. The experiences provided should differ in outward appearance while retaining the same basic conceptual structure. The provision of multiple experiences (not the same experience many times), using a variety of materials, is designed to promote abstraction of the mathematical concept. When children are given opportunities to see a concept in different ways and under different conditions, they are more likely to perceive that concept irrespective of its concrete embodiment. For example, the regrouping procedures (ten ones exchanged for one ten, ten tens for one hundred, and so forth) used in the process of adding two numbers is independent of the type of materials used. The teacher could therefore use tongue depressors, rubber bands, chips, an abacus, or multibase arithmetic blocks to illustrate the regrouping process. When exposed to a number of seemingly different tasks that are identical in structure, children will tend to abstract the similar elements from their experiences. It is not the performance of any one of the individual tasks that is the mathematical abstraction but the ultimate realization of their similarity. Children thus will realize that it is not the particular material that is important but the exchange process because that is the variable common to all embodiments. This process is known as mathematical *abstraction*.

2.3.3 The Mathematical Variability Principle

This third principle suggests that the generalization of a mathematical concept is enhanced when variables irrelevant to that concept are systematically varied while keeping the relevant variables constant. For example, if one is interested in promoting an understanding of the term *parallelogram*, this principle suggests that it is desirable to vary as many of the irrelevant attributes as possible. In this instance, the size of angles, the length of sides, and the position on the paper should be varied while keeping the only crucial attributes a four-sided figure with opposite sides parallel-intact. Many persons erroneously believe that squares and rectangles are *not* parallelograms. This misconception has resulted because the appropriate mathematical variables (in this case angle size) had not been manipulated when they were taught the concept. There are many other similar examples. Each is an endorsement of the arguments made by Dienes to provide consciously for the systematic manipulating of irrelevant variables in our instruction. Dienes suggests that the two variability principles be used in concert with one another. They are intended to promote the complementary processes of abstraction and generalization, both of which are crucial aspects of conceptual development.

2.3.4 The Constructivity Principle

Dienes identifies two kinds of thinkers: the constructive thinker and the analytical thinker. He roughly equates the constructive thinker with Piaget's concrete operational stage and the analytical thinker with Piaget's formal operational stage of cognitive development.

The constructivity principle states simply that "construction should always precede analysis." It is analogous to the assertion that children should be allowed to develop their concepts in a global intuitive manner beginning with their own experiences. According to Dienes, these constructive experiences should form the cornerstone on which all mathematics learning is based. At some future time, attention can be directed toward the analysis of what has been constructed; however, Dienes points out that it is not possible to analyze what is not yet there in some concrete form. One major problem in schools is the fact that many children are asked to abstract mathematical ideas before they have the opportunity to experience them in concrete form. A common result is rote learning. The constructivity principle, although simplistic in concept, if implemented, would have profound implications for change in mathematics classrooms.

2.3.5 Summary and Implications

The unifying theme of these four principles is undoubtedly that of stressing the importance of learning mathematics by means of direct interaction with the environment. Dienes is continually implying that mathematics learning is not a spectator sport and, as such, requires a very active type of physical and mental involvement on the part of the learner. In addition to stressing the environmental role in effective conceptual learning, Dienes addresses in his two variability principles the problem of providing for individualized learning rates and learning styles. His constructivist principle aligns itself closely with the work of Piaget and suggests a developmental approach to the learning of mathematics that is ordered so as to coincide with the various stages of intellectual development. The following are some implications of Dienes's work:

- The whole-class (or large-group) lesson would be greatly deemphasized in order to accommodate individual differences in ability and interests;
- Individual and small-group activities would be used concomitantly because it is not likely that more than two to four children would be ready for the same experience at the same point in time;
- The role of the teacher would include exposition as well as being a facilitator;
- The role of students would be expanded. They would assume a greater degree of responsibility for their own learning;
- The newly defined learning environment would create new demands for additional sources of information and direction. The creation of a learning laboratory containing a large assortment of materials and other conceptual amplifiers such as computers would be a natural result of serious consideration of Dienes's ideas (Reys & Post, 1973).

II.4 ADVANCED LEVEL MATHEMATICAL CURRICULUM

2.4.1 Background to curriculum review

The motive of reviewing the Mathematics syllabus of advanced level was to ensure that the syllabus is responsive to the needs of the learner and to shift from objective and knowledge-based learning to competence-based learning. Emphasis in the review is put more on skills and **competences** and the coherence within the existing content by benchmarking with syllabi elsewhere with best practices. The new Mathematics syllabus guides the interaction between the teacher and the learners in the learning processes and highlights the **competences** a learner should acquire during and at the end of each unit of learning. Learners will have the opportunity to apply Mathematics in different contexts, and see its important in daily life. Teachers help the learners appreciate the relevance and benefits for studying this subject in advanced level. The new Mathematics syllabus is prepared for all science combinations with Mathematics as core subject where it has to be taught in seven periods per week.

2.4.2 Rationale of teaching and learning Mathematics

1. **Mathematics and society**

Mathematics plays an important role in society through abstraction and logic, counting, calculation, measurement, systematic study of shapes and motion. It is also used in natural sciences, engineering, medicine, finance and social sciences. The applied mathematics like statistics and probability play an important role in game theory, in the national census process, in scientific research, etc. In addition, some cross-cutting issues such as financial awareness are incorporated into some of the Mathematics units to improve social and economic welfare of Rwandan society.

Mathematics is key to the Rwandan education ambition of developing a knowledge-based and technology-led economy since it provide to learners all required knowledge and skills to be used in different learning areas. Therefore, Mathematics is an important subject as it supports other subjects. This new curriculum will address gaps in the current Rwanda Education system which lacks of appropriate skills and attitudes provided by the current education system.

2. Mathematics and learners

Learners need enough basic mathematical **competences** to be effective members of Rwandan society including the ability to estimate, analyze, interpret statistics, assess probabilities, and read the commonly used mathematical representations and graphs. Therefore, Mathematics equips learners with knowledge, skills and attitudes necessary to enable them to succeed in an era of rapid technological growth and socio-economic development. Mastery of basic Mathematical ideas and calculations makes learners being confident in problem-solving. It enables the learners to be systematic, creative and self confident in using mathematical language and techniques to reason; think critically; develop imagination, initiative and flexibility of mind. In this regard, learning of Mathematics needs to include practical problem-solving activities with opportunities for students to plan their own investigations in order to develop their mathematical competence and confidence. As new technologies have had a dramatic impact on all aspects of life, wherever possible in Mathematics, learners should gain experience of a range of ICT equipment and applications.

3. Competences

Competence is defined as the ability to perform a particular task successfully, resulting from having gained an appropriate combination of knowledge, skills and attitudes. The Mathematics syllabus gives the opportunity to learners to develop different **competences**, including the generic **competences**. Basic **competences** are addressed in the stated broad subject competences and in objectives highlighted year on year basis and in each of units of learning. The generic **competences**, basic competences that must be emphasized and reflected in the learning process are briefly described below and teachers will ensure that learners are exposed to tasks that help the learners acquire the skills.

2.4.3 Generic competences and Values

Critical and problem solving skills: Learners use different techniques to solve mathematical problems related to real life situations. They are engaged in mathematical thinking; they construct, symbolize, apply and generalize mathematical ideas. The acquisition of such skills will help learners to think imaginatively and broadly to evaluate and find solutions to problems encountered in all situations.

Creativity and innovation: The acquisition of such skills will help learners to take initiatives and use imagination beyond knowledge provided to generate new ideas and construct new concepts. Learners improve these skills through Mathematics contest, Mathematics competitions, etc

Research: This will help learners to find answers to questions basing on existing information and concepts and to explain phenomena basing on findings from information gathered.

Communication in official languages: Learners communicate effectively their findings through explanations, construction of arguments and drawing relevant conclusions. Teachers, irrespective of not being teachers of language, will ensure the proper use of the language of instruction by learners which will help them to communicate clearly and confidently and convey ideas effectively through speaking and writing and using the correct language structure and relevant vocabulary.

Cooperation, inter personal management and life skills: Learners are engaged in cooperative learning groups to promote higher achievement than do competitive and individual work.

This will help them to cooperate with others as a team in whatever task assigned and to practice positive ethical moral values and respect for the rights, feelings and views of others. Perform practical activities related to environmental conservation and protection. Advocating for personal, family and community health, hygiene and nutrition and Responding creatively to the variety of challenges encountered in life.

Lifelong learning: The acquisition of such skills will help learners to update knowledge and skills with minimum external support and to cope with evolution of knowledge advances for personal fulfillment in areas that need improvement and development

2.4.4 Broad mathematics competences

During and at the end of learning process, the learner can:

- Develop clear, logical, creative and coherent thinking;
- Master basic mathematical concepts and use them correctly in daily life problem solving;
- Express clearly, comprehensibly, correctly and precisely in verbal and/or in written form all the reasons and calculations leading to the required result whenever finding a solution to any given exercise;
- Master the presented mathematical models and to identify their applications in his/her environment;
- Arouse learner's mathematical interest and research curiosity in theories and their applications;
- Use the acquired mathematical concepts and skills to follow easily higher studies (Colleges, Higher Institutions and Universities);
- Use acquired mathematical skills to develop work spirit, team work, self-confidence and time management without supervision;
- Use ICT tools to explore Mathematics (**examples**: calculators, computers, mathematical software, etc);
- Demonstrate a sense of research, curiosity and creativity in their areas of study.

2.4.5 Mathematics and developing competences

The national policy documents based on national aspirations identify some ”**basic competences”** alongside the **“Generic competences”** that will develop higher order thinking skills and help student learn subject content and promote application of acquired knowledge and skills.

Through observations, constructions, using symbols, applying and generalizing mathematical ideas, and presentation of information during the learning process, the learner will not only develop deductive and inductive skills but also acquire cooperation and communication, critical thinking and problem solving skills. This will be realized when learners make presentations leading to inferences and conclusions at the end of learning unit. This will be achieved through learner group work and cooperative learning which in turn will promote interpersonal relations and teamwork.

The acquired knowledge in learning Mathematics should develop a responsible citizen who adapts to scientific reasoning and attitudes and develops confidence in reasoning independently. The learner should show concern of individual attitudes, environmental protection and comply with the scientific method of reasoning. The scientific method should be applied with the necessary rigor, intellectual honesty to promote critical thinking while systematically pursuing the line of thought.

The selection of types of learning activities must focus on what the learners are able to demonstrate such **competences** throughout and at the end of the learning process.

2.4.6 Pedagogical approach

The change to a competence-based curriculum is about transforming learning, ensuring that learning is deep, enjoyable and habit-forming.

1. **Role of the learner**

In the competence-based syllabus, the learner is the principal actor of his/her education. He/she is not an empty bottle to fill. Taking into account the initial capacities and abilities of the learner, the syllabus lists under each unit, the activities of the learner and they all reflect appropriate engagement of the learner in the learning process. The teaching- learning processes will be tailored towards creating a learner friendly environment basing on the capabilities, needs, experience and interests. Therefore, the following are some of the roles or the expectations from the learners:

- Learners construct the knowledge either individually or in groups in an active way. From the learning theory, learners move in their understanding from concrete through pictorial to abstract. Therefore, the opportunities should be given to learners to manipulate concrete objects and to use models;

- Learners are encouraged to use hand-held calculator. This stimulates mathematics as it is really used, both on job and in scientific applications. Frequent use of calculators can enhance learners' understanding and mastering of arithmetic;
- Learners work on one competence at a time in form of concrete units with specific learning objectives broken down into knowledge, skills and attitude.
- Learners will be encouraged to do research and present their findings through group work activities;
- A learner is cooperative: learners work in heterogeneous groups to increase tolerance and understanding;
- Learners are responsible for their own participation and ensure the effectiveness of their work;
- Help is sought from within the group and the teacher is asked for help only when the whole group agrees to ask a question;
- The learners who learn at a faster pace do not do the task alone and then the others merely sign off on it;
- Participants ensure the effective contribution of each member, through clear explanation and argumentation to improve the English literacy and to develop sense of responsibility and to increase the self-confidence, the public speech ability, etc.

2. Role of the teacher

In the competence-based syllabus, the teacher is a facilitator, organizer, advisor, a conflict solver, ... The specific duties of the teacher in a competence-based approach are the following:

- He/she is a facilitator, his/her role is to provide opportunities for learners to meet problems that interest and challenge them and that, with appropriate effort, they can solve. This requires an elaborated preparation to plan the activities, the place they will be carried, the required assistance;
- He/she is an organizer: his/her role is to organize the learners in the classroom or outside and engage them through participatory and interactive methods through the learning processes as individuals, in pairs or in groups. To ensure that the learning is personalized, active and participative , co-operative the teacher must identify the needs of the learners, the nature of the learning to be done, and the means to shape learning experiences accordingly;
- He/she is an advisor: he/she provides counseling and guidance for learners in need. He/she comforts and encourages learners by valuing their contributions in the class activities;
- He/she is a conflict-solver: most of the activities competence-based are performed in groups. The members of a group may have problems such as attribution of tasks; they should find useful and constructive the intervention of the teacher as a unifying element.

- He/she is ethical and preaches by examples by being impartial, by being a role-model, by caring for individual needs, especially for slow learners and learners with physical impairments, through a special assistance by providing remedial activities or reinforcement activities. One should notice that this list is not exhaustive.

3. **Special needs education and inclusive approach**

All Rwandans have the right to access education regardless of their different needs. The underpinnings of this provision would naturally hold that all citizens benefit from the same menu of educational programs. The possibility of this assumption is the focus of special needs education. The critical issue is that we have persons/ learners who are totally different in their ways of living and learning as opposed to the majority. The difference can either be emotional, physical, sensory and intellectual learning challenged traditionally known as mental retardation.

These learners equally have the right to benefit from the free and compulsory basic education in the nearby ordinary/mainstream schools. Therefore, the schools' role is to enroll them and also set strategies to provide relevant education to them. The teacher therefore is requested to consider each learner's needs during teaching and learning process. Assessment strategies and conditions should also be standardized to the needs of these learners. Detailed guidance for each category of learners with special education needs is provided for in the guidance for teachers.

2.4.7 Assessment approach

Assessment is the process of evaluating the teaching and learning processes through collecting and interpreting evidence of individual learner's progress in learning and to make a judgment about a learner's achievements measured against defined standards. Assessment is an integral part of the teaching learning processes. In the new competence-based curriculum assessment must also be competence-based; whereby a learner is given a complex situation related to his/her everyday life and asked to try to overcome the situation by applying what he/she learned. Assessment will be organized at the following levels: School-based assessment, District examinations, National assessment and National examinations.

1. **Types of assessments**

   Formative assessments: Formative assessment helps to check the efficiency of the process of learning. It is done within the teaching/learning process. Continuous assessment involves formal and informal methods used by schools to check whether learning is taking place. When a teacher is planning his/her lesson, he/she should establish criteria for performance and behavior changes at the beginning of a unit. Then at the end of every unit, the teacher should ensure that all the learners have mastered the stated key unit **competences** basing on the criteria stated, before going to the next unit. The teacher will assess how well each learner masters both the subject and the generic **competences** described in the syllabus and from this, the teacher will gain a picture of the all-round progress of the learner. The teacher will use one or a combination of the following:

- Observation;
- Pen and paper;
- Oral questioning.

Summative assessments: When assessment is used to record a judgment of a competence or performance of the learner, it serves a summative purpose. Summative assessment gives a picture of a learner's competence or progress at any specific moment. The main purpose of summative assessment is to evaluate whether learning objectives have been achieved and to use the results for the ranking or grading of learners, for deciding on progression, for selection into the next level of education and for certification. This assessment should have an integrative aspect whereby a student must be able to show mastery of all **competences**.

It can be internal school based assessment or external assessment in the form of national examinations. School based summative assessment should take place once at the end of each term and once at the end of the year. School summative assessment average scores for each subject will be weighted and included in the final national examinations grade. School based assessment average grade will contribute a certain percentage as teachers gain more experience and confidence in assessment techniques and in the third year of the implementation of the new curriculum it will contribute 10% of the final grade, but will be progressively increased. Districts will be supported to continue their initiative to organize a common test per class for all the schools to evaluate the performance and the achievement level of learners in individual schools. External summative assessment will be done at the end of P6, S3 and S6.

Record keeping: This is gathering facts and evidence from assessment instruments and using them to judge the student's performance by assigning an indicator against the set criteria or standard. Whatever assessment procedures used shall generate data in the form of scores which will be carefully be recorded and stored in a portfolio because they will contribute for remedial actions, for alternative instructional strategy and feed back to the learner and to the parents to check the learning progress and to advice accordingly or to the final assessment of the students.

This portfolio is a folder (or binder or even a digital collection) containing the student's work as well as the student's evaluation of the strengths and weaknesses of the work. Portfolios reflect not only work produced (such as papers and assignments), but also it is a record of the activities undertaken over time as part of student learning. . Besides, it will serve as a verification tool for each learner that he/she attended the whole learning before he/she undergoes the summative assessment for the subject.

Item writing in summative assessment: Before developing a question paper, a plan or specification of what is to be tested or examined must be elaborated to show the units or topics to be tested on, the number of questions in each level of Bloom's taxonomy and the marks allocation for each question. In a competence based curriculum, questions from higher levels of Bloom's taxonomy should be given more weight than those from knowledge and comprehension level. Before developing a question paper, the item writer must ensure that the test or examination questions are tailored towards competence based assessment by doing the following:

- Identify topic areas to be tested on from the subject syllabus;
- Outline subject-matter content to be considered as the basis for the test;
- Identify learning outcomes to be measured by the test;
- Prepare a table of specifications;
- Ensure that the verbs used in the formulation of questions do not require memorization or recall answers only but testing broad **competences** as stated in the syllabus.

Structure and format of the examination: There will be one paper in Mathematics at the end of Primary 6. The paper will be composed by two sections, where the first section will be composed with short answer items or items with short calculations which include the questions testing for knowledge and understanding, investigation of patterns, quick calculations and applications of Mathematics in real life situations. The second section will be composed with long answer items or answers with demonstrations, constructions, high level reasoning, analysis, interpretation and drawing conclusions, investigation of patterns and generalization. The items for the second section will emphasize on the mastering of Mathematics facts, the understanding of Mathematics concepts and its applications in real life situations. In this section, the assessment will find out not only what skills and facts have been mastered, but also how well learners understand the process of solving a mathematical problem and whether they can link the application of what they have learned to the context or to the real life situation. The Time required for the paper is three hours (3hrs).

The following topic areas have to be assessed: Trigonometry; algebra; analysis; linear algebra; geometry; statistics and probability. Topic areas with more weight will have more emphasis in the second section where learners should have the right to choose to answer 3 items out of 5.

The wider range of learning in the new curriculum means that it is necessary to think again about how to share learners' progress with parents. A single mark is not sufficient to convey the different expectations of learning which are in the learning objectives. The most helpful reporting is to share what students are doing well and where they need to improve.

2.4.8 Resources

1. **Materials needed for implementation**

The following list shows the main materials/equipments needed in the learning and teaching process:

- Materials to encourage group work activities and presentations: Computers (Desk tops &lab tops) and projectors; Manila papers and markers;
- Materials for drawing & measuring geometrical figures/shapes and graphs: Geometric instruments, ICT tools such as geogebra, Microsoft student ENCARTA, etc
- Materials for enhancing research skills: Textbooks and internet (the list of the textbooks to consult is given in the reference at the end of the syllabus and those books can be found in printed or digital copies).
- Materials to encourage the development of Mathematical models: scientific calculators, Math type, Matlab, etc
- The technology used in teaching and learning of Mathematics has to be regarded as tools to enhance the teaching and learning process and not to replace teachers.

2. **Human resource**

The effective implementation of this curriculum needs a joint collaboration of educators at all levels. Given the material requirements, teachers are expected to accomplish their noble role as stated above. On the other hand school head teachers and directors of studies are required to make a follow-up and assess the teaching and learning of this subject due to their profiles in the schools. These combined efforts will ensure bright future careers and lives for learners as well as the contemporary development of the country.

In a special way, the teacher of Mathematics at ordinary level should have a firm understanding of mathematical concepts at the level he/she teaches. He/she should be qualified in Mathematics and have a firm ethical conduct. The teacher should possess the qualities of a good facilitator, organizer, problem solver, listener and adviser. He/she is required to have basic skills and competence of guidance and counseling because students may come to him or her for advice.

The teacher of Mathematics should have the following skills, values and qualities:

- Engage learners in variety of learning activities;
- Use multiple teaching and assessment methods;
- Adjust instruction to the level of the learners;
- Have creativity and innovation the teaching and learning process;
- Be a good communicator and organizer;
- Be a guide/ facilitator and a counselor;
- Manifest passion and impartial love for children in the teaching and learning process ;
- Make useful link of Mathematics with other Subjects and real life situations;
- Have a good master of the Mathematics Content;
- Have good classroom management skills.

2.4.9 Syllabus Units

1. **Presentation of the structure of the syllabus units**

Mathematics subject is taught and learnt in advanced level of secondary education as a core subject, **i.e** in S4, S5 and S6 respectively. At every grade, the syllabus is structured in Topic Areas, sub-topic Areas where applicable and then further broken down into Units to promote the uniformity, effectiveness and efficiency of teaching and learning Mathematics. The units have the following elements:

1. Unit is aligned with the Number of Lessons;
2. Each Unit has a Key Unit Competence whose achievement is pursued by all teaching and learning activities undertaken by both the teacher and the learners;
3. Each Unit Key Competence is broken into three types of Learning Objectives as follows:
    - Type I: Learning Objectives relating to Knowledge and Understanding (Type I Learning Objectives are also known as Lower Order Thinking Skills or LOTS);
    - Type II and Type III: These Learning Objectives relate to acquisition of skills, Attitudes and Values (Type II and Type III Learning Objectives are also known as Higher Order Thinking Skills or HOTS) – These Learning Objectives are actually considered to be the ones targeted by the present reviewed curriculum.
4. Each Unit has a Content which indicates the scope of coverage of what to be taught and learnt in line with stated learning objectives
5. Each Unit suggests a non exhaustive list of Learning Activities that are expected to engage learners in an interactive learning process as much as possible (learner-centered and participatory approach);
6. Finally, each Unit is linked to Other Subjects, its Assessment Criteria and the Materials (or Resources) that are expected to be used in teaching and learning process. The Mathematics syllabus for ordinary level has got;
7. Topic Areas: Trigonometry, Algebra, Analysis, Linear algebra, Geometry, Statistics and Probability and these topic areas are found in each of the three grades of the advanced level which are S4, S5 and S6. As for units, they are 16 in S4, 10 in S5 and 9 in S6

2.4.10 Mathematics program for secondary four, five and six

1 KEY COMPETENCE AT THE END OF SECONDARY FOUR

After completion of secondary 4, the mathematics syllabus will help the learner to:

- Use the trigonometric concepts and formulas in solving problem related to trigonometry;
- Think critically and analyze daily life situations efficiently using mathematical logic concepts and infer conclusion;
- Model and solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities;
- Represent graphically simple numerical functions;

- Perform operations on linear transformation and solve problems involving geometric transformations;
- Determine algebraic representations of lines, straight lines and circles in the 2 dimension;
- Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation;
- Use matrices and determinants of order 2 to solve systems of linear equations and to define transformations of 2 dimension;
- Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation;
- Use counting techniques and concepts of probability to determine the probability of possible outcomes of events occurring under equally likely assumptions

2. KEY COMPETENCES AT THE END OF SECONDARY FIVE

After completion of secondary 5, the mathematics syllabus will help the learner to:

- Extend the use of the trigonometric concepts and transformation formulas to solve problems involving trigonometric equations, inequalities and or trigonometric identities;
- Use arithmetic, geometric and harmonic sequences, including convergence to understand and solve problems arising in various context;
- Solve equations involving logarithms or exponentials and apply them to model and solve related problems;
- Study and to represent graphically a numerical function;
- Apply theorems of limits and formulas to solve problems involving differentiation including optimization, etc
- Study linear dependence of vectors of 3 IR and perform operations on linear transformations of 3 IR using vectors;
- Extend the use of matrices and determinants to order 3 to solve problems in various contexts;
- Use algebraic representations of lines, spheres and planes in 3D space and solve related problems;
- Extend the understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines;
- Solve problems using Bayes theorem and data to make decisions about likelihood and risk.

3. KEY COMPETENCES AT THE END OF SECONDARY SIX

After completion of secondary 6, the mathematics syllabus will help the learners to:

- Extend understanding of sets of numbers to complex numbers;

- Solve polynomial equations in the set of complex numbers and solve related problems in physics, etc;
- Extend the use of concepts and definitions of functions to determine the domain of logarithmic and exponential functions;
- Use integration as the inverse of differentiation and as the limit of a sum and apply them to finding area and volumes to solve various practical problems;
- Use differential equations to solve related problems that arise in a variety of practical contexts;
- Relate the sum and the intersection of subspaces of a vector space by the dimension formula;
- Determine the kernel and the image of a linear transformation and use the results to infer the properties of a linear transformation;
- Transform a matrix to its equivalent form using elementary row operations;
- Determine algebraic representations of conics in the plane and use them to represent and interpret physical phenomena;
- Use probability density functions of a random variable to model events, including binomial, Poisson and Normal distributions.

Chap III: MATHEMATICAL ACTIVITIES/LESSONS

The mathematical activities are the sets of all activities in the classroom, assessment, mathematical lessons, mathematical references, etc

Activity

1. *What/Where/When do you assess in mathematics?*
2. *Give different levels of Bloom's taxonomy.*
3. *Give the general format of Lesson plan?*
4. *Develop a Mathematical subject on your choice and does a lesson plan on the developed subject by respecting all parts of lesson plan?*
5. *Develop deeply one of the Ordinary Level Mathematics topics on your choice?*
6. *What do you understand by scheme of work and give the content of scheme of work?*
7. *By using the format of scheme of work: Do the scheme of work of one term of senior 3 Ordinary Level based on REB Curriculum.*

II1.1 Implementation of assessment in Mathematics and Science

The following aspects are recommended to consider for the teachers to exercise better assessment of their learners;

1. **About the time of assessment**
    - At the beginning of the lesson to determine the prerequisite of learners;

- After each step of a lesson to get the feedback;
- At the end of every topic or subtopic to assess the level of understanding;
- At the end of a day or a week as homework to create the research skills;
- Whenever teachers want to record and compare the results of learners.

2. **About the attitude of the assessor**
   - Assess the background of learners before tackling on the contents of the lesson;
   - Create in the mind of learners that assessment is a way of improving the teaching and learning process;
   - Appreciate/praise good answers from learners;
   - Encourage learners to respond even when the answers are not correct;
   - Have a time to comment on learners' performance in form of advice, encouragement and a wish for better performance;
   - Place all learners in the same conditions while assessing;
   - Promote the frame such as learners can correct one another mistake;
   - Make learners used to oral, written and practical questions;
   - Record the results and return the papers to learners on time.
3. **About the quality of question paper**
   - Set standard question paper: i.e. With many and various questions starting from simple to complex;
   - Avoid ambiguous questions;
   - Avail the marking scheme and the question paper before the starting of the test;
   - Locate marks, timing, and instructions on the question paper;
   - Collect and supply all needed materials for practical examination and test them before the coming of learners;
4. **About area to assess**
   - **Cognitive:** Intend to assess the thinking skill of the learners;
   - **Psychomotor:** intend to assess the ability of handling and the presentation of learners' findings;
   - **Affective:** intend to assess the attitude of the learners during the learning process and their interaction with each other.

III.2 Levels of Bloom's Taxonomy

Level 1: REMEMBER: Retrieve relevant knowledge from long term memory

Categories and Cognitive Names	Alternative Names	Definitions and Examples
1.1 Recognizing	Identifying	Locating knowledge in long term memory that is consistent with presented material. **E.G: -**Recognize the types of muscles

		-State Newton's second law
1.2 Recalling	Retrieving	Retrieving relevant knowledge from long term memory **E.G:-**Recall the first law of Gregor Mendel

Level 2: UNDERSTAND: Construct meaning from instructional messages, including oral, Written, and graphic communication

Categories and Cognitive Names	**Alternative Names**	**Definitions and Examples**
2.1 Interpreting	Paraphrasing, Clarifying, Representing, Translating	Changing from one form of representing (text) to another (diagram). Interpreting of a diagram of water cycle.
2.2 Exemplifying	Illustrating, Instantiating	Finding a specific example or illustration of a concept or principles. Give example of Archimedes' principle.
2.3 Classifying	Categorizing, Subsuming	Determining that something belongs to category. **E.G:-**Classify the organic compound -Classify the simple machine(levers) in their different classes
2.4 Summarizing	Abstracting, Generalizing	Abstracting general theme or major points. **E.G:** Write a summary of the evolution event portrayed on a videotape.
2.5 Inferring	Concluding, Extrapolating, Predicting	Drawing a logical conclusion from presented information. **E.G:** Putting different material in plastic beaker which containing water
2.6 Comparing	Contrasting, Mapping, Matching	Detecting correspondences between two ideas, objects, and the like **E.G:** Matching an element of group A and group B

Level 3: APPLY: Carry out or use a procedure in a given situation

Categories and Cognitive Names	Alternative Names	Definitions and Examples
3.1 Executing	Carrying out	Applying the procedure to a familiar task. **E.G:** Divide one whole number by another whole number, both with multiple digits.
3.2 Implementing	Using	Applying a procedure to unfamiliar task **E.G:** Use the Newton's second law in situations in which it is appropriate.

Level 4: ANALYZE: Break material into its constituent parts and determine how the parts relate to one another and to an overall structure or purpose

Categories and Cognitive Names	Alternative Names	Definitions and Examples
4.1 Differentiating	Discriminating, Distinguishing, Focusing, selecting	Distinguishing relevant from irrelevant parts or important from unimportant parts of presented material **E.G:** Distinguish relevant and irrelevant numbers in a mathematical word problem.
4.2 Organizing	Finding coherence, integrating, outlining, parsing, Structuring	Determining how the elements fit or function within a structure. **E.G:** Set up an experiment of filtration by using the given materials.
4.3 Attributing	Deconstructing	Determine a point of view, bias, value, intent underlying presented scientific theory. **E.G:** Determine the point of view of Bohr in chemistry.

Level 5: EVALUATION: Make judgment based on criteria and standards

Categories and Cognitive Names	Alternative Names	Definitions and Examples
5.1 checking	Coordinating, Detecting, Monitoring,	Detecting inconsistencies or fallacies within a process or product, determining whether a process or product has internal consistency, detecting the effectiveness of a procedure as it is implemented.

	Testing	**E.G:** Determine if a scientist's conclusion follow from observed data.

Level 6: CREATE: Put elements together to form a coherent or functional whole, reorganize element into pattern or structure

Categories and Cognitive Names	Alternative Names	Definitions and Examples
6.1 Generating	Hypothesizing	Coming up with alternative hypotheses based on criteria. **E.G:** Generate hypotheses to account for an observed phenomenon
6.2 Planning	Designing	Devising a procedure for accomplishing some task. **E.G:** Plan a research paper on a given topic.
6.3 Producing	Constructing	Investing a product. **E.G:** Improvise a telescope

III.3 Rationale

During the baseline survey in May 2008, learners and teachers of secondary schools in Rwanda indicated mathematical subjects have direct application in our everyday life or philosophical theory of the subject.

III.4 General Objectives or learning objectives

By the ends of the module/course students should be able to

- Formulating and solving by using different methods the simultaneous equations;
- Teaching and operating by using vectors.

III.5 Key unit competence

After completion of secondary 4, the mathematics syllabus will help the learner to:

- Use the trigonometric concepts and formulas in solving problem related to trigonometry;
- Think critically and analyze daily life situations efficiently using mathematical logic concepts and infer conclusion;
- Model and solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities;

After completion of secondary 5, the mathematics syllabus will help the learner to:

- Extend the use of the trigonometric concepts and transformation formulas to solve problems involving trigonometric equations, inequalities and or trigonometric identities;
- Use arithmetic, geometric and harmonic sequences, including convergence to understand and solve problems arising in various context;

After completion of secondary 6, the mathematics syllabus will help the learners to:

- Extend understanding of sets of numbers to complex numbers;
- Solve polynomial equations in the set of complex numbers and solve related problems in physics, etc;
- Extend the use of concepts and definitions of functions to determine the domain of logarithmic and exponential functions;
- Use integration as the inverse of differentiation and as the limit of a sum and apply them to finding area and volumes to solve various practical problems;

But each performed course must have the key competence after it completion (Cfr Mathematical curriculum).

III.6 Knowledge; understanding; skills; attitude and values

1. **Skills, Values and quality of teachers**

- Engage learners in variety of learning activities;
- Use multiple teaching and assessment methods;
- Adjust instruction to the level of the learners;

2. **Knowledge and understanding**
   - Teachers and learners may have knowledge about mathematics and science activities;
   - Teachers may have knowledge and skills in management/conduction/supervision of group activities;
   - The documentation and libraries will help learners and teachers in course content;
   - Laboratory activities and practical activities will be a key word in science knowledge and understanding;

III.7 General Format of lesson plan

<u>The general format of new lesson plan 1</u>

Names	**School**	**Term**	**Date**	**Lesson No**

Topic: …………………………………………………

Sub topic: ……………………………………………

Class: third year of Ordinary Level

Duration: x minutes

Number of Students: x students

Type of special educational needs and number of learners: x learner with disability (Specify the type of disability and number of learners with disability

Rationale…………………………………………………………………………………

Objectives……………………………………………………………………………….

Key unit competence…………………………………………………………………

Knowledge&understanding ……………………………………………………………

Skills………………………………………………………………………………………

Attitudes&values……………………………………………………………………..

Plan for this class (Location: In/ Outside)……………………………………………….

Prerequisite knowledge…………………………………………………………………

Teaching and learning materials…………………………………………………………

Reference…………………………………………………………………………………

Step/Duration	**Teachers' activities**	**Learners' activities**	**Learning points**	**Competences and cross cutting issues to be addressed**
Introduction (y min)				Knowledge: Skills: Attitude and Values:
Body of the Lesson **(x min)**				Knowledge: Skills: Attitude and Values: Cross cutting issues:
Summary/				Knowledge:

Conclusion **(z min)**				Skills: Attitude and Values:
Evaluation **(t min)**				

<u>The general format of new lesson plan 2</u>

Names	**School**	**Term**	**Date**	**Lesson No**

Topic: ………………………………………………

Sub topic: ……………………………………………

Class: third year of Ordinary Level

Duration: x minutes

Number of Students: x students

Type of special educational needs and number of learners: x learner with disability (Specify the type of disability and number of learners with disability

Rationale……………………………………………………………………………………

Objectives…………………………………………………………………………………….

Key unit competence………………………………………………………………………

Knowledge&understanding ………………………………………………………………

Skills…………………………………………………………………………………………

Attitudes&values……………………………………………………………………………..

Plan for this class (Location: In/ Outside)……………………………………………….

Prerequisite knowledge……………………………………………………………………

Teaching and learning materials…………………………………………………………

Reference……………………………………………………………………………………

Step/Duration	Teachers/Learner's activities	Learning points	Competences and cross cutting issues to be addressed
Introduction (y min)			Knowledge: Skills: Attitude and Values:
Body of the Lesson (x min)			Knowledge: Skills: Attitude and Values: Cross cutting issues:
Summary/ Conclusion (z min)			Knowledge: Skills: Attitude and Values:
Evaluation (t min)			

N.B: Format 1 and 2 are the same except in the case that we increase the number of columns by separating teachers' activities and learners' activities. The learning activities are a part that consists with mathematical theories that the teacher provides in place of learners.

III.8 Matrices, determinants and Simultaneous equations

1. **Definition**

The simultaneous equation is the equation of the form $\begin{cases} a\,x_1 + bx_2 = c \\ d\,x_1 + ex_2 = f \end{cases}$ where x_1 *and* x_2 are unknown variables and $a, b, c, d. e, and\ f$ are real numbers/constants. The solution consist of finding the values x_1 *and* x_2 that satisfy both equations, the solution is written by $S = \{(x_1, x_2)\}$. More advanced we can have a simultaneous equation of more than two equations and more than two unknown variables.

Example $\begin{cases} a\,x_1 + bx_2 + cx_3 = d \\ a'x_1 + b'x_2 + c'x_3 = d' \\ a''x_1 + b''x_2 + c''x_3 = d'' \end{cases}$

When we use efficiently, problem solving has advantages such as:

- To help learners develop new skills and knowledge for themselves and feel responsible for their own learning;

- To provide learners with an opportunity to apply their knowledge and to see that their knowledge has some real world applications.

2. Specific objectives

Learner should be able to:

- Define linear simultaneous equation;
- Form linear simultaneous equation;
- Solve linear simultaneous equation;
- Solve world problems using linear simultaneous equation.

3. Knowledge and understanding

- Define operations on matrices of order 3;
- Illustrate the properties of determinants of matrices of order 3;
- Show that a square matrix of order 3 is invertible or not;
- Define a linear transformation in 3D by a matrix;
- Define and perform operations on linear transformations of IR^3;
- Express analytically the inverse of an isomorphism of IR^3;
- Discuss with respect to a parameter the solutions of a system of three linear equations in three unknowns.

4. Skills

- Perform operations on matrices of order 3;
- Calculate the determinants of matrices of order 3;
- Explain using determinant whether a matrix of order 3 is invertible or not;
- Determine the inverse of a matrix of order 3;
- Reorganize data into matrices;
- Determine the matrix of a linear transformation in 3D;
- Apply matrices to solve related problems;
- Apply the concepts of linear transformation to determine and perform various operations on linear transformations of IR^3 ;
- Use the properties of linear transformation of IR^3 to construct the analytic expression of the inverse of an isomorphism of IR^3;
- Use Cramer's rule to solve a system of three linear equations in three unknowns;
- Apply properties of determinants to solve problems related to matrices of order 3

5. Attitudes and values

- Appreciate the importance of matrices of order 3 and their determinant s in organizing data and solving related problems;
- Appreciate the importance of operations on linear transformations of IR^3 and their properties

6. Formulation of simultaneous equation

Three examples of formulating simultaneous equations are enough: one example for 2×2 equation and 3×3 eqaution

7. Methods of solving simultaneous equation

- **Numerical methods**

- **Graphical methods**
- **Substitution methods**
- **Gauss elimination methods**
- **Cramer's methods**
- **Gauss-Jordan Methods**
- **Inverse matrix methods**

8. Notes and Examples of simultaneous Equations

Let consider the system of n-equations with n-unknown variables

$$\begin{cases} a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = d_1 \\ b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_1x_n = d_2 \\ \quad\quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad . \\ z_1x_1 + z_2x_2 + z_3x_3 + \cdots + z_nx_n = d_n \end{cases}$$ this system can be written in term of matrices as

$$\begin{pmatrix} a_1 & a_2 & a_3 & \dots & a_n \\ b_1 & b_2 & b_3 & \dots & b_n \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ z_1 & z_2 & z_3 & \dots & z_n \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ . \\ . \\ . \\ d_n \end{pmatrix}$$ $More\ general\ the\ system\ is\ written\ AX = D$ where

$$A = \begin{pmatrix} a_1 & a_2 & a_3 & \dots & a_n \\ b_1 & b_2 & b_3 & \dots & b_n \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ z_1 & z_2 & z_3 & \dots & z_n \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix} \ and \ D = \begin{pmatrix} d_1 \\ d_2 \\ . \\ . \\ . \\ d_n \end{pmatrix}$$ the condition of having solution to the system is that the determinant of the matrix A must be different from 0 i.e $\det(A) \neq 0$ and the set of solution is written in the form of $S = \{(x_1, x_2, x_3, \dots, x_n)\} \in IR^n$

There are various methods for solving the system of equation such as

- **Gauss elimination method**: We use different operation on rows and columns of the system to reduce the system on the matrix A to a triangular matrix. The new $AX = D$ gives solution to the system;
- **Cramer's method**: consist of finding the determinant of the system Δ and the determinant that correspond to various unknown variables $\Delta_{x_1}, \Delta_{x_2}, \Delta_{x_3}, \dots, \Delta_{x_n}$. The corresponding solution is $S = \{(x_1, x_2, x_3, \dots, x_n)\} = \left\{(\frac{\Delta_{x1}}{\Delta}, \frac{\Delta_{x2}}{\Delta}, \frac{\Delta_{x3}}{\Delta}, \dots, \frac{\Delta_{xn}}{\Delta})\right\}$;
- **Inverse matrix method**: consist of finding the solution $S = \{(x_1, x_2, x_3, \dots, x_n)\}$ by using inverse matrix as $AX = D \Rightarrow X = A^{-1}D$ where A^{-1} is the inverse matrix of A;
- **Gauss - Jordan method**: consist of using different operations on rows and columns on reducing the matrix $(A|D)$ to $(I|S)$ Where I in the unity/canonical matrix of the same dimension with A and S is the solution matrix

$$(A|D) \Rightarrow (I|S) = \left(\begin{array}{ccccc|c} a_1 & a_2 & a_3 & \dots & a_n & d_1 \\ b_1 & b_2 & b_3 & \dots & b_n & d_2 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ z_1 & z_2 & z_3 & \dots & z_n & d_n \end{array}\right) \Rightarrow \left(\begin{array}{ccccc|c} 1 & 0 & 0 & \dots & 0 & x_1 \\ 0 & 1 & 0 & \dots & 0 & x_2 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1 & x_n \end{array}\right)$$

<u>If n=2:</u> (2 equations and 2 unknown variables) the system becomes $\begin{cases} a_1x_1 + a_2x_2 = d_1 \\ b_1x_1 + b_2x_2 = d_2 \end{cases}$ and in term of matrices becomes $\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \Rightarrow AX = D$ which is the example of the given system on $\begin{cases} 5x_1 + 3x_2 = 59 \\ -6x_1 + 31x_2 = 47 \end{cases} \Rightarrow \begin{pmatrix} 5 & 3 \\ -6 & 31 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 59 \\ 47 \end{pmatrix} \Rightarrow AX = D\ where$

$A = \begin{pmatrix} 5 & 3 \\ -6 & 31 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\ and\ D = \begin{pmatrix} 59 \\ 47 \end{pmatrix}$

Because det(A)=$\begin{vmatrix} 5 & 3 \\ -6 & 31 \end{vmatrix} = 155 + 18 = 173$, then the system admit a solution

$S = \{(x_1, x_2)\} \in IR^2$

- **Gauss elimination method** $\begin{cases} 5x_1 + 3x_2 = 59 \\ -6x_1 + 31x_2 = 47 \end{cases} \Rightarrow \begin{cases} 5x_1 + 3x_2 = 59 \\ 0x_1 + 173x_2 = 589 \end{cases} \Rightarrow \begin{pmatrix} 5 & 3 \\ 0 & 173 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 59 \\ 589 \end{pmatrix} \Rightarrow 173x_2 = 589 \Rightarrow x_2 = \frac{589}{173}$
  If $x_2 = \frac{589}{173}$ then $5x_1 + 3x_2 = 59$ implies that $5x_1 + 3\frac{589}{173} = 59 \Rightarrow 5x_1 = 59 - 3\frac{589}{173} \Rightarrow x_1 = \frac{1688}{173}$. The solution is $S = \{(x_1, x_2)\} = \left\{\left(\frac{1688}{173}, \frac{589}{173}\right)\right\}$
- **Cramer's method**
  $\begin{cases} 5x_1 + 3x_2 = 59 \\ -6x_1 + 31x_2 = 47 \end{cases} \Rightarrow \begin{pmatrix} 5 & 3 \\ -6 & 31 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 59 \\ 47 \end{pmatrix}$
  The determinant of the system is $\Delta = \begin{vmatrix} 5 & 3 \\ -6 & 31 \end{vmatrix} = 155 + 18 = 173$
  The determinant of x_1 variable is
  $\Delta_{x_1} = \begin{vmatrix} 59 & 3 \\ 47 & 31 \end{vmatrix} = 59 * 31 - 3 * 47 = 1829 - 141 = 1688$
  The determinant of x_1 variable is
  $\Delta_{x_1} = \begin{vmatrix} 5 & 59 \\ -6 & 47 \end{vmatrix} = 5 * 47 + 6 * 59 = 235 + 354 = 589$
  The solution is $S = \{(x_1, x_2)\} = \left\{\left(\frac{\Delta_{x_1}}{\Delta}, \frac{\Delta_{x_2}}{\Delta}\right)\right\} = \left\{\left(\frac{1688}{173}, \frac{589}{173}\right)\right\}$
- **Inverse matrix method**
  $\begin{cases} 5x_1 + 3x_2 = 59 \\ -6x_1 + 31x_2 = 47 \end{cases} \Rightarrow \begin{pmatrix} 5 & 3 \\ -6 & 31 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 59 \\ 47 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ -6 & 31 \end{pmatrix}^{-1} \begin{pmatrix} 59 \\ 47 \end{pmatrix}$
  Note: If $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ is a matrix such that det(A)$\neq 0$, $A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$ is its inverse matrix.
  $A = \begin{pmatrix} 5 & 3 \\ -6 & 31 \end{pmatrix}$ is a matrix such that $\det(A) = \begin{vmatrix} 5 & 3 \\ -6 & 31 \end{vmatrix} = 155 + 18 = 173$ then

$$A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix} = \frac{1}{173}\begin{pmatrix} 31 & -3 \\ 6 & 5 \end{pmatrix} = \begin{pmatrix} \frac{31}{173} & \frac{-3}{173} \\ \frac{6}{173} & \frac{5}{173} \end{pmatrix}$$

Thus $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{31}{173} & \frac{-3}{173} \\ \frac{6}{173} & \frac{5}{173} \end{pmatrix}\begin{pmatrix} 59 \\ 47 \end{pmatrix} = \begin{pmatrix} \frac{31}{173} \times 59 - 47 \times & \frac{3}{173} \\ \frac{6}{173} \times 59 + 47 \times & \frac{5}{173} \end{pmatrix} = \begin{pmatrix} \frac{1688}{173} \\ \frac{589}{173} \end{pmatrix}$

Therefore, The solution is $S = \{(x_1, x_2)\} = \left\{\left(\frac{1688}{173}, \frac{589}{173}\right)\right\}$

- **Gauss-Jordan's method :** we use echelon matrix

  $(A|D) = \begin{pmatrix} 5 & 3 & | & 59 \\ -6 & 31 & | & 47 \end{pmatrix}$ $R_1' = \frac{R_1}{5}$ *and* $R_2' = 6R_1 + 5R_2 \Rightarrow \begin{pmatrix} 1 & \frac{3}{5} & | & \frac{59}{5} \\ 0 & 173 & | & 589 \end{pmatrix}$

  $R_2' = \frac{R_2}{173} \Rightarrow \begin{pmatrix} 1 & \frac{3}{5} & | & \frac{59}{5} \\ 0 & 1 & | & \frac{589}{173} \end{pmatrix}$ $R_1' = 3\frac{R_2}{5} \Rightarrow \begin{pmatrix} 1 & 0 & | & \frac{1688}{173} \\ 0 & 1 & | & \frac{589}{173} \end{pmatrix} = (I|S)$

  The solution is $S = \{(x_1, x_2)\} = \left\{\left(\frac{1688}{173}, \frac{589}{173}\right)\right\}$

- **The solution of** $\begin{cases} 5x_1 + 3x_2 = 59 \\ -6x_1 + 31x_2 = 47 \end{cases}$ **can be obtained by using graphical method:**

  We draw the two straight lines $(5x_1 + 3x_2 = 59 \text{ } and \text{ } -6x_1 + 31x_2 = 47)$ in Cartesian plane and the intersection of them is then the solution.

<u>If n=3:</u> (3 equations and 3 unknown variables) the system becomes $\begin{cases} a_1x_1 + a_2x_2 + a_3x_3 = d_1 \\ b_1x_1 + b_2x_2 + b_3x_3 = d_2 \\ c_1x_1 + c_2x_2 + c_3x_3 = d_3 \end{cases}$

In term of matrix the system becomes $\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \Rightarrow AX = D$ which is the example of the given system on $\begin{cases} 3x_1 + 15x_2 + 10x_3 = 79 \\ 2x_1 + 16x_2 + 9x_3 = 65 \\ -11x_1 + 5x_2 - x_3 = 2 \end{cases} \Rightarrow \begin{pmatrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} =$ $\begin{pmatrix} 79 \\ 65 \\ 2 \end{pmatrix} \Rightarrow AX = D$ where

$A = \begin{pmatrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ *and* $D = \begin{pmatrix} 79 \\ 65 \\ 2 \end{pmatrix}$. By using Sarrus's rule we can find the value of det(A) $\Rightarrow \left|\begin{matrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{matrix}\right| \begin{matrix} 3 & 15 \\ 2 & 16 \\ -11 & 5 \end{matrix}$

$= 3 * 16 * (-1) + 15 * 9 * (-11) + 10 * 2 * 5 - (-11) * 16 * 10 - 5 * 9 * 3 - (-1) * 2 * 15 = -48 - 1485 + 100 + 1760 - 135 + 30 = 222$

Because det(A)$\neq 0$ then the system admit the solution of the form $S = \{(x_1, x_2, x_3)\} \in IR^3$

- **Gauss elimination method** $\begin{cases} 3x_1 + 15x_2 + 10x_3 = 79 \\ 2x_1 + 16x_2 + 9x_3 = 65 \\ -11x_1 + 5x_2 - x_3 = 2 \end{cases} \Rightarrow \begin{cases} 3x_1 + 15x_2 + 10x_3 = 79 \\ 0x_1 - 18x_2 - 7x_3 = -37 \\ 0x_1 + 180x_2 + 107x_3 = 875 \end{cases}$ *by*

$$R_2' = 2R_1 - 3R_2 \text{ and } R_3' = 11R_1 + 3R_3$$

$\Rightarrow \begin{cases} 3x_1 + 15x_2 + 10x_3 = 79 \\ 0x_1 - 18x_2 - 7x_3 = -37 \\ 0x_1 + 0x_2 + 37x_3 = 505 \end{cases} \Rightarrow \begin{pmatrix} 3 & 15 & 10 \\ 0 & -18 & -7 \\ 0 & 0 & 37 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 79 \\ -37 \\ 505 \end{pmatrix} \Rightarrow$

$37x_3 = 505 \Rightarrow x_3 = \frac{505}{37} = \frac{3030}{222}$

$-18x_2 - 7x_3 = -37 \Rightarrow -18x_2 - 7\left(\frac{505}{37}\right) = -37 \Rightarrow -18x_2 = -37 + \frac{3535}{63} \Rightarrow x_2 = \frac{-722}{222}$

$3x_1 + 15x_2 + 10x_3 = 79 \Rightarrow 3x_1 = 79 - 15x_2 - 10x_3$

$\Rightarrow 3x_1 = 79 - 15\left(-\frac{505}{63}\right) - 10\left(\frac{10918}{1134}\right) x_1 = \frac{-644}{222}$. The set of solution is

$S = \{(x_1, x_2, x_3)\} = \left\{\left(\frac{\Delta_{x_1}}{\Delta}, \frac{\Delta_{x_2}}{\Delta}, \frac{\Delta_{x_3}}{\Delta}\right)\right\} = \left\{\left(\frac{-644}{222}, \frac{-722}{222}, \frac{3030}{222}\right)\right\}$

- **Cramer's method**

$\begin{cases} 3x_1 + 15x_2 + 10x_3 = 79 \\ 2x_1 + 16x_2 + 9x_3 = 65 \\ -11x_1 + 5x_2 - x_3 = 2 \end{cases} \Rightarrow \begin{pmatrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 79 \\ 65 \\ 2 \end{pmatrix}$

Det(A)=$\begin{vmatrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{vmatrix} = 222$ is the determinant of the system.

$\Delta_{x_1} = \begin{vmatrix} 79 & 15 & 10 \\ 65 & 16 & 9 \\ 2 & 5 & -1 \end{vmatrix} = 79\begin{vmatrix} 16 & 9 \\ 5 & -1 \end{vmatrix} - 65\begin{vmatrix} 15 & 10 \\ 5 & -1 \end{vmatrix} + 2\begin{vmatrix} 15 & 10 \\ 16 & 9 \end{vmatrix}$

$= 79(-16 - 45) - 65(-15 - 20) + 2(135 - 160) = -644$. The determinant that corresponds to x_1

$\Delta_{x_2} = \begin{vmatrix} 3 & 79 & 10 \\ 2 & 65 & 9 \\ -11 & 2 & -1 \end{vmatrix} = 3\begin{vmatrix} 65 & 9 \\ 2 & -1 \end{vmatrix} - 2\begin{vmatrix} 79 & 10 \\ 2 & -1 \end{vmatrix} - 11\begin{vmatrix} 79 & 10 \\ 65 & 9 \end{vmatrix} = -722$. The determinant that corresponds to x_2

$\Delta_{x_3} = \begin{vmatrix} 3 & 15 & 79 \\ 2 & 16 & 65 \\ -11 & 5 & 2 \end{vmatrix} = 3\begin{vmatrix} 16 & 65 \\ 5 & 2 \end{vmatrix} - 2\begin{vmatrix} 15 & 79 \\ 5 & 2 \end{vmatrix} - 11\begin{vmatrix} 15 & 79 \\ 16 & 65 \end{vmatrix} = 3030$; The determinant that corresponds to x_3. Therefore the setoff solution is

$S = \{(x_1, x_2, x_3)\} = \left\{\left(\frac{\Delta_{x_1}}{\Delta}, \frac{\Delta_{x_2}}{\Delta}, \frac{\Delta_{x_3}}{\Delta}\right)\right\} = \left\{\left(\frac{-644}{222}, \frac{-722}{222}, \frac{3030}{222}\right)\right\}$

- **Inverse matrix method**

Let $\begin{cases} 3x_1 + 15x_2 + 10x_3 = 79 \\ 2x_1 + 16x_2 + 9x_3 = 65 \\ -11x_1 + 5x_2 - x_3 = 2 \end{cases} \Rightarrow \begin{pmatrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 79 \\ 65 \\ 2 \end{pmatrix}$ be the system, then $AX = D \Rightarrow X = A^{-1}D$

We need to find the inverse matrix of $A = \begin{pmatrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{pmatrix}$ that is A^{-1} and the matrix is invertible if it determinant is different from 0. Det(A)=222$\neq 0$.

$A^{-1} = \frac{1}{\det(A)}[adjoint\ (A)]^T$. Let calculate the adjoint matrix of A.

$$\text{Adj(A)}=\begin{pmatrix} \begin{vmatrix} 16 & 9 \\ 5 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 9 \\ -11 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 16 \\ -11 & 5 \end{vmatrix} \\ -\begin{vmatrix} 15 & 10 \\ 5 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 10 \\ -11 & -1 \end{vmatrix} & -\begin{vmatrix} 3 & 15 \\ -11 & 5 \end{vmatrix} \\ \begin{vmatrix} 15 & 10 \\ 16 & 9 \end{vmatrix} & -\begin{vmatrix} 3 & 10 \\ 2 & 9 \end{vmatrix} & \begin{vmatrix} 3 & 15 \\ 2 & 16 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -61 & -97 & 186 \\ 65 & 107 & -180 \\ -25 & -7 & 18 \end{pmatrix}$$

$[adjoint\ (A)]^T = \begin{pmatrix} -61 & 65 & -25 \\ -97 & 107 & -7 \\ 186 & -180 & 18 \end{pmatrix}, henceA^{-1} = \begin{pmatrix} \frac{-61}{222} & \frac{65}{222} & \frac{-25}{222} \\ \frac{-97}{222} & \frac{107}{222} & \frac{-7}{222} \\ \frac{186}{222} & \frac{-180}{222} & \frac{18}{222} \end{pmatrix}$ .The set of solution is

$$X = A^{-1}D \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{-61}{222} & \frac{65}{222} & \frac{-25}{222} \\ \frac{-97}{222} & \frac{107}{222} & \frac{-7}{222} \\ \frac{186}{222} & \frac{-180}{222} & \frac{18}{222} \end{pmatrix} \begin{pmatrix} 79 \\ 65 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{-4819+4225-50}{222} \\ \frac{-7663+6955-14}{222} \\ \frac{14694-11700+36}{222} \end{pmatrix} = \begin{pmatrix} \frac{-644}{222} \\ \frac{-722}{222} \\ \frac{3030}{222} \end{pmatrix}$$

Thus $S = \{(x_1, x_2, x_3)\} = \left\{(\frac{-644}{222}, \frac{-722}{222}, \frac{3030}{222})\right\}$

- **Gauss-Jordan's method :** we use echelon matrix

Let $\begin{cases} 3x_1 + 15x_2 + 10x_3 = 79 \\ 2x_1 + 16x_2 + 9x_3 = 65 \\ -11x_1 + 5x_2 - x_3 = 2 \end{cases} \Rightarrow \begin{pmatrix} 3 & 15 & 10 \\ 2 & 16 & 9 \\ -11 & 5 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 79 \\ 65 \\ 2 \end{pmatrix}$ This can be reduced to

$$\begin{pmatrix} 3 & 15 & 10 & & 79 \\ 2 & 16 & 9 & | & 65 \\ -11 & 5 & -1 & & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 15 & 10 & & 79 \\ 0 & -18 & -7 & | & -37 \\ 0 & 180 & 107 & & 875 \end{pmatrix}$$

$Where\ R_2' = 2R_1 - 3R_2\ and\ R_3' = 11R_1 + 3R_3$

$$\Rightarrow \begin{pmatrix} 3 & 15 & 10 & & 79 \\ 0 & -18 & -7 & | & -37 \\ 0 & 0 & 37 & & 505 \end{pmatrix}\ where\ R_3' = 10R_2 + R_3$$

$$\Rightarrow\begin{pmatrix}1 & 5 & \frac{10}{3} & | & \frac{79}{3}\\ 0 & 1 & \frac{7}{18} & | & \frac{37}{18}\\ 0 & 0 & 1 & & \frac{505}{37}\end{pmatrix} \textit{Where } R_1' = \frac{R_1}{3}, R_2' = \frac{R_2}{18} \textit{ and } R_3' = \frac{R_3}{37}$$

$$\Rightarrow\begin{pmatrix}1 & 0 & \frac{25}{18} & | & \frac{289}{18}\\ 0 & 1 & \frac{7}{18} & | & \frac{37}{18}\\ 0 & 0 & 1 & & \frac{505}{37}\end{pmatrix} \textit{Where } R_1' = -5R_2 + R_1$$

$$\Rightarrow\begin{pmatrix}1 & 0 & 0 & | & \frac{-644}{222}\\ 0 & 1 & 0 & | & \frac{-722}{222}\\ 0 & 0 & 1 & & \frac{505}{37}\end{pmatrix} \textit{Where } R_2' = -\frac{7}{18}R_3 + R_1 \textit{ and } R_1' = -\frac{25}{18}R_3 + R_2$$

Thus the solution is $S = \{(x_1, x_2, x_3)\} = \left\{(\frac{-644}{222}, \frac{-722}{222}, \frac{3030}{222})\right\}$

N.B: The provided solutions and notations may correspond to the level of students. The facilitator must provide the methods that correspond to the level of students. For example, don't use matrices notation on students of senior three.

6. Example of lesson plan on simultaneous equation

Topic: Simultaneous equation

Sub topic: Solving simultaneous equation of 3 D

Class: fifth year of advanced level

Duration: 40 minutes

Number of Students: 45

Type of special educational needs and number of learners: 2 learners with disability (One has no right leg and one has no left arm)

Rationale

Simultaneous equations are very useful in our daily life such as in shopping, in sharing. They are also used in Physics, Economics, etc

Objectives

By the end of this lesson learners should be able to solve simultaneous equation by using inverse matrix method.

Key unit competence

Extend the use of matrices and determinants to order 3 to solve problems in various contexts.

Knowledge & understanding

- Show the matrices notation of a system of equation;
- Show that a square matrix of order 3 is invertible or not;
- Discuss with respect to a parameter the solutions of a system of three linear equations in three unknown variables.

Skills

- Calculate the determinants of matrices of order 3;
- Explain using determinant whether a matrix of order 3 is invertible or not;
- Determine the inverse of a matrix of order 3;
- Use Cramer's rule to solve a system of three linear equations in three unknowns.

Attitudes & values

- Appreciate the importance of matrices of order 3 and their determinant s in organizing data and solving related problems;

Plan for this class (Location: In/ Outside): In classroom

Prerequisite knowledge

For proper understanding of the topic, learners should have a good knowledge of matrices presentations and operations, determinant calculation, formulation of systems of equation and calculation of inverse matrix.

Teaching and learning materials

Chalks, chalk board,

Reference

Pioneer mathematics XII, Pure mathematics, secondary school linear algebra, etc

<table>
<tr><th>Step/Duration</th><th>Teacher/Learners’ activities</th><th>Learning points</th><th>Competences and cross cutting issues to be addressed</th></tr>
<tr><td>Introduction (5 min)</td><td>The teacher makes the revision on forming linear simultaneous equation

Alice, John and Mary went to Supermarket for shopping. Alice brought 3 pens, 2 books and a pencil for 41 $; John brought 2 pens, a book and 2 pencils for 29$ and Mary brought 2 pens, 2 books and 2 pencils for 44$

Express this situation by using mathematical symbols, and form simultaneous equation</td><td>The formed simultaneous equation is
$\begin{cases} 3\,x_1 + 2x_2 + x_3 = 41 \ (1) \\ 2x_1 + x_2 + 2x_3 = 29 \ \ (2) \\ 2x_1 + 2x_2 + 2x_3 = 44 \ \ (3) \end{cases}$
By letting $x_1{:}\, pens;\ x_2{:}\, books$ &
$x_3{:}\, Pencils$
$X = \{(x_1, x_2, x_3)\}$ is solution;
The system is $AX = D \Rightarrow X = A^{-1}D$ where A^{-1} is the inverse matrix of A:Matrix of the system</td><td>Knowledge: Formulation

Skills: System notation

Attitude and Values: Appreciate the shopping example</td></tr>
<tr><td>Body of the Lesson

(27 min)</td><td>The teacher divides learners into 7 groups of 5 students.
The learners choose group leader and secretary.
The teacher distributes the worksheet to learners/ Write the activity on board.
Learners do the activity in groups.
The teacher gives instructions to the learners about the activity.
Learners present their findings on board.
Teacher leads discussion</td><td>Step 1:Matrices notation of system
$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 41 \\ 29 \\ 44 \end{pmatrix}$
Step 2:Determinant of A
Det(A)=$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{vmatrix} = -4$ The matrix A is invertible
Step 3:Let calculate the ad joint matrix of A: Adj(A)= matrix of minors and signature
$Adj(A) = \begin{pmatrix} -2 & 0 & 2 \\ -2 & 4 & -2 \\ 3 & -4 & -1 \end{pmatrix}$
Step 4: Let calculate inverse matrix $A^{-1} = \frac{1}{Det\,(A)}[Adj(A)]^T$
$A^{-1} = \frac{1}{-4}\begin{pmatrix} -2 & -2 & 3 \\ 0 & 4 & -4 \\ 2 & -2 & -1 \end{pmatrix}$
Step 5: The solution: $X = A^{-1}D$</td><td>Knowledge: Inverse matrix calculation, Solving system by using inverse matrix.

Skills: Supervision of group activities, Group discussion and learners presentation of findings

Attitude and Values: Appreciate the learners” discussions, Appreciate the learners ability in problem solving.</td></tr>
</table>

		becomes $$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\frac{1}{-4}\begin{pmatrix}-2&-2&3\\0&4&-4\\2&-2&-1\end{pmatrix}\begin{pmatrix}41\\29\\44\end{pmatrix}$$ $$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2\\15\\5\end{pmatrix}$$ **Step 6:** Test the solution by replacing solutions in system 3*2+2*15+5=41 OK in (1) 2*2+15+2*5=29 OK in (2) 2*2+2*15+2*5=44 OK in (3) **Step 7:** The set of solution is X={(2,15, 5)}is the solution of the system $\begin{cases}3\,x_1+2x_2+x_3=41\\2x_1+x_2+2x_3=29\\2x_1+2x_2+2x_3=44\end{cases}$	
Summary/ Conclusion (3 min)	The teacher helps learners to summarize and to make the conclusion	The set of solution is X={(2,15, 5)} The pens is sold on 2$, the book on 15$ and the pencil on 5$	**Knowledge**: solution Formulation **Skills**: solution interpretation **Attitude and Values**: Appreciate the solut^{o}
Evaluation (5 min)	The teacher provides the exercises to learners and the assignment is given to students; Exercises are given on board as homework	1. $\begin{cases}x_1+2x_2+5x_3=10\\x_1-x_2-x_3=-2\\2x_1+3x_2-x_3=-11\end{cases}$	

		2. $\begin{cases} x_1 - 2x_2 = 10 \\ 2x_1 + x_2 + 3x_3 = 8 \\ -2x_2 + x_3 = 7 \end{cases}$	

III.9 Statistics

This part will consign with examples of mathematical subjects (theories and examples) of the advanced level curricula that a teacher may have before teaching.

1. **Statistics:** this is the important chapter of senior three. It is always assessed during Ordinary level National Examination.

Descriptive statistics is the discipline of quantitatively describing the main features of a collection of information, or the quantitative description itself. Descriptive statistics are distinguished from inferential statistics (or inductive statistics), in that descriptive statistics aim to summarize a sample, rather than use the data to learn about the population that the sample of data is thought to represent. This generally means that descriptive statistics, unlike inferential statistics, are not developed on the basis of probability theory. Even when a data analysis draws its main conclusions using inferential statistics, descriptive statistics are generally also presented. For example in a paper reporting on a study involving human subjects, there typically appears a table giving the overall sample size, sample sizes in important subgroups.

Some measures that are commonly used to describe a data set are measures of central tendency and measures of variability or dispersion. Measures of central tendency include the mean, median, mode, quartiles, deciles and percentiles while measures of variability include the standard deviation , variance, range, variation coefficient, inter quartile interval, absolute mean deviation.

statistical inference is the process of drawing conclusions from data that are subject to random variation, Initial requirements of such a system of procedures for inference and induction are that the system should produce reasonable answers when applied to well-defined situations and that it should be general enough to be applied across a range of situations. Inferential statistics are used to test hypotheses and make estimations using sample data.

The outcome of statistical inference may be an answer to the question "what should be done next?", where this might be a decision about making further experiments or surveys, or about drawing a conclusion before implementing some organizational or governmental policy.

a. **Mean:** The mean is one of the measures of central tendency parameter; it shows the average value of the given statistical numbers.

The needs mean is the statistical mean that can be calculated by $\overline{\boldsymbol{X}} = \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{fi}\, \boldsymbol{X_i}$ Where $\boldsymbol{fi} =$ frequencies. If X_o is the assumed mean the statistical mean can be calculated by $\overline{\boldsymbol{X}} = \boldsymbol{X_0} + \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{fi}\,(\boldsymbol{X_i} - \boldsymbol{X_0})$

There are different types of the means the statistical mean ($\overline{\boldsymbol{X}} = \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{fi}\, \boldsymbol{X_i}$, $\boldsymbol{fi} =$ frequencies) **,** the arithmetic mean ($\boldsymbol{AM} = \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{X_i}$ **),** the geometric mean [GM=$(\prod_{i=1}^{n} Xi)^{\frac{1}{n}}$] and the harmonic mean is ($HM = \sum_{i=1}^{n} \frac{1}{X_i}$)

b. **Mode :** Is the statistical number with highest frequency, it is the most repeated number.
c. **Median:** The median is the number of statistical data which gives a half of number on its left and another half of number on its right; it is the number which is located in the central of statistical series Xi. Let n be the total number of the statistical numbers, the median has the position $\frac{\boldsymbol{X_{n+1}}}{\mathbf{2}}$ If n is odd and $\frac{X_{\frac{n}{2}} + X_{\frac{n}{2}+1}}{2}$ if n is even**.**
d. **Standard deviation**: In statistics and probability theory, the **standard deviation** (**SD**) (represented by the Greek letter sigma **σ**) shows how much variation or dispersion from the average exists. A low standard deviation indicates that the data points tend to be very close to the mean (also called expected value); a high standard deviation indicates that the data points are spread out over a large range of values.
Mathematically, the standard deviation is defined as square root of variance.
If the statistical number are expressed in function of frequencies the variance is given by $Var(x) = \frac{1}{n}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{f_i}(\boldsymbol{Xi} - \overline{\boldsymbol{X}})^{\mathbf{2}} = \frac{1}{n}\sum_{i=1}^{n} f_i X_i^2 - \overline{\boldsymbol{X}}^{\mathbf{2}}$ This implies that the standard deviation can be calculated by SD(x)=$\sqrt{Var(x)} = \sqrt{\frac{1}{n}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{f_i}(\boldsymbol{Xi} - \overline{\boldsymbol{X}})^{\mathbf{2}}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} f_i X_i^2 - \overline{\boldsymbol{X}}^{\mathbf{2}}}$
e. **Range:** is the difference between the highest value of the statistical series and the smallest value of the statistical series: $X_n - X_1$
f. **Bar chart:** Is the diagram of bars that shows the statistical numbers in function of frequencies. There can be bar chart of frequencies and of cumulative frequencies.
g. **Histogram:** Is the diagram of grouped data that shows class interval of statistical number in function of frequencies. There can be histogram of frequencies and of cumulative frequencies. The class interval can be obtained by using Sturge’s formula 1+(3.3 log k) or by using Yule’s formula: 2.5 $\sqrt[4]{n}$

2. Knowledge and understanding

- Define mean, mode and median;
- Define the variance, standard deviation and the coefficient of variation;
- Analyze and interpret critically data and infer conclusion.

3. Skills

- Determine the measure of position of a given statistical series;
- Determine the measures of dispersion of a given statistical series;
- Apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data;

- Express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.

4. Attitudes and values

- Appreciate the importance of measures of position in the interpretation of data;
- Appreciate the importance of measures of dispersion in the interpretation of data;
- Show concern on how to use the standard deviation and variation coefficient as measure of variability of data;

QUESTION: Given the table below

Xi	3	4	2
Yi	4	6	2

a. Find the variance of Xi and that of Yi

b. Find the covariance of (Xi, Yi)

c. Find the correlation coefficient

d. Find the regression lines

NUMERICAL APPLICATION

Let consider the following table

Xi	fi	Cfi	fi Xi	X_i^2	$fi\,X_i^2$	
3	1	1	3	9	9	
5	2	3	10	25	50	
6	1	4	6	36	36	
10	1	5	10	100	100	
12	5	10	60	144	720	
21	2	12	42	441	882	
23	1	13	23	529	529	
45	1	14	45	2025	2025	
	14		199		4351	Summation

Remember that cumulative frequencies Cf_i=$f_1 + f_2 + f_3 + \cdots + f_n = \sum_{i=1}^{n} f_i$

a. Mean

$\boldsymbol{\overline{X} = \frac{1}{n}\sum_{i=1}^{n} fi\, X_i = \frac{1}{14} * 199 = 14.21486}$

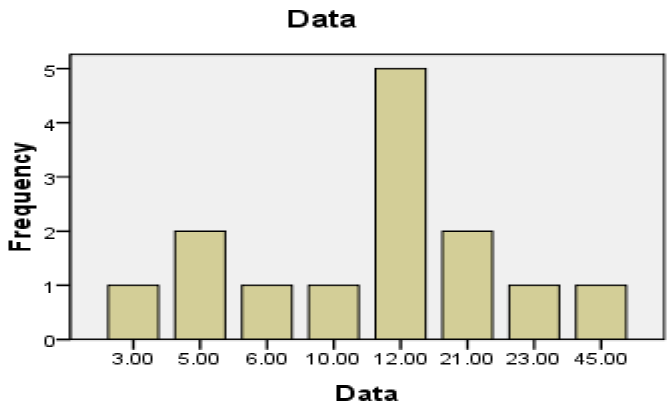
b. Mode : The mode is 12 with $f_i = 5$

c. Median: n=14 is even, then $Med = \frac{X_{\frac{n}{2}} + X_{\frac{n}{2}+1}}{2} = \frac{X_7+X_8}{2} = \frac{12+12}{2} = 12$

d. Standard deviation: SD(x)=$\sqrt{Var(x)} = \sqrt{\frac{1}{n}\boldsymbol{\sum_{i=1}^{n} f_i(Xi - \overline{X})^2}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} f_i X_i^2 - \boldsymbol{\overline{X}^2}}$

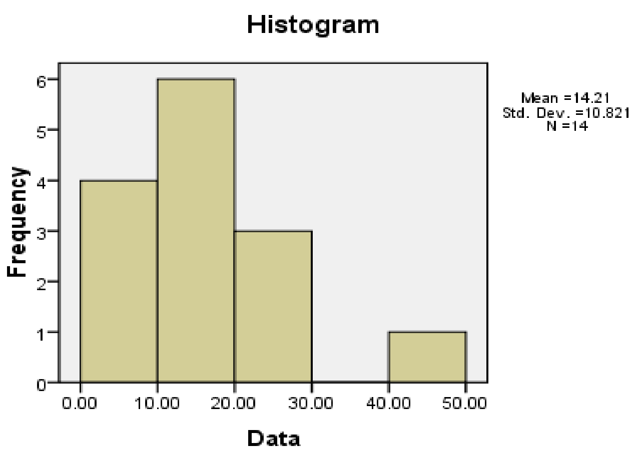
$$SD(x) = \sqrt{\frac{1}{14} * 4351 - \mathbf{14.21486}^{\,2}} = \sqrt{310.7857 - 202.0622} = \sqrt{108.7235} =$$

e. Bar chart: The following chart shows the bar chart of frequencies in function of data or statistical data.



f. Histogram: the following chart shows the histogram of frequencies in function of data; data are 5 class intervals.

$$0 - 10\,, 10 - 20\,, 20 - 30\,, 30 - 40 \; and \; 40 - 50$$



N.B: The bar chart and the histogram were obtained by using SPSS program.

Example of lesson plan on dispersion parameters

Topic: Dispersion parameters

Sub topic: Calculating variation coefficient of statistical series

Class: fourth year of advanced level

Duration: 40 minutes

Number of Students: 35

Type of special educational needs and number of learners: There is no learner with disability

Rationale

Dispersion parameters are very useful in our daily life such as in counting and prediction of population, in surveying of population and households. They are also used in Economics and in National Institute of Statistics, etc

Objectives

By the end of this lesson learners should be able to calculate and to interpret the variation coefficient of a statistical series.

Key unit competence

- Extend the understanding, analysis and interpretation of dispersion parameters to statistical series;

Knowledge & understanding

- Define the variance, standard deviation and the coefficient of variation;
- Analyze and interpret critically data and infer conclusion

Skills

- Determine the measures of dispersion of a given statistical series;
- Express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.

Attitudes & values

- Show concern on how to use the standard deviation and variation coefficient as measure of variability of data.

Plan for this class (Location: In/ Outside): In classroom

Prerequisite knowledge

For proper understanding of the topic, learners should have a good knowledge of statistical series, mean, range and inter-quartile interval.

Teaching and learning materials

Chalks, chalk board,

Reference

Pioneer mathematics XI, Pure mathematics, secondary school element of statistics, etc

Step/Duration	Teacher/Learners' activities	Learning points	Competences and cross cutting issues to be addressed
Introduction (5 min)	The teacher makes the revision on forming statistical series; The teacher makes the revision on the position and on the dispersion parameters Mean, Variance and standard deviation by using Statistical notations; • Kagabo had got the followings marks in 16 different days out of 10: 4,5,6,6,5,4,6,5,5,6,7,7,7,5,3,8	Let X_i be the marks of Kagabo and let f_i be the marks' repetition. The statistical table combining X_i and f_i becomes $\begin{array}{ccccccc} X_i & 3 & 4 & 5 & 6 & 7 & 8 \\ f_i & 1 & 2 & 5 & 4 & 3 & 1 \end{array}$ Mean: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} fi\ X_i$ Variance: $Var(x) = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{f_i(Xi - \bar{X})^2} = \frac{1}{n}\sum_{i=1}^{n} f_i X_i^2 - \boldsymbol{\bar{X}^2}$ Standard deviation SD(x)=$\sqrt{Var(x)}$ Variation coefficient $VC(X) = \frac{SD(X)}{\bar{X}}$	**Knowledge**: Formulation **Skills**: System notation **Attitude and Values**: Appreciate the shopping example
Body of the Lesson **(27 min)**	• The teacher divides learners into 7 groups of 5 students. • The learners choose group leader and secretary. • The teacher distributes the worksheet to learners/ Write the activity on board. • Learners do the activity in groups. • The teacher gives instructions to the learners about the activity. • Learners present their findings on board. • Teacher leads discussion	**Step 1:**Number of statistical series $n = \sum_{i=1}^{n} f_i = 16$ **Step 2:**Table related to formula $\begin{array}{ccccc} X_i & f_i & X_i f_i & X_i^2 & f_i X_i^2 \\ 3 & 1 & 3 & 9 & 9 \\ 4 & 2 & 8 & 16 & 32 \\ 5 & 5 & 25 & 25 & 125 \\ 6 & 4 & 24 & 36 & 144 \\ 7 & 3 & 21 & 49 & 147 \\ 8 & 1 & 8 & 64 & 64 \\ & 16 & 89 & & 521 \end{array}$	**Knowledge**: Inverse matrix calculation, Solving system by using inverse matrix. **Skills**: Supervision of group activities, Group discussion and learners presentation of findings; ability to talk publically **Attitude and Values**: Appreciate the learners" discussions, Appreciate the learners ability in

		Step 3: Let calculate the mean $$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} fi\, X_i = \frac{89}{16} = 5.56$$ **Step 4:** Let calculate the variance : $Var(x) = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{f_i}(\boldsymbol{Xi} - \bar{\boldsymbol{X}})^2 = \frac{1}{n}\sum_{i=1}^{n} f_i X_i^2 - \bar{\boldsymbol{X}}^2 = \frac{\boldsymbol{521}}{\boldsymbol{16}} - \boldsymbol{5.56^2} = \boldsymbol{1.65}$ **Step 5:** let calculate the standard deviation SD(x)=$\sqrt{Var(x)} = \sqrt{1.65} = 1.28$ **Step 6:**Let calculate the variation coefficient $$VC(X) = \frac{SD(X)}{\bar{\boldsymbol{X}}} = \frac{1.28}{5.56} = 0.2302$$	problem solving.
Summary/ **Conclusion** **(3 min)**	• Teacher harmonize; • Learners give their points of view and make a summary.	• The variation coefficient of 4,5,6,6,5,4,6,5,5,6,7,7,7,5,3,8 is 23.02 %, it shows how statistical data are spread out of the mean.	**Knowledg**e: solution Formulation **Skills**: solution interpretation **Attitude and Values**: Appreciate the solut°
Evaluation **(5 min)**	The teacher provides the exercises to learners and the assignment is given to students; Exercises are given on board as homework	Calculate the variation coefficient of 11,12,13,14,15,11,12,12,14,14,15,12,14, 15,14,15,13,12,14,15,16,15,13,11,11,15	

III.10 DISCRETE RANDOM VARIABLE

1 INTRODUCTION

In this chapter, we shall first consider experiments with a finite number of possible outcomes $m_1, m_2, m_3, \ldots, m_n$ For example, we roll a dice and the possible outcomes are $\{1,2,3,4,5,6\}$ .Let toss a coin with possible outcome H (heads) and T (tails).

It is frequently useful to be able to refer to an outcome of an experiment. For example, we might want to write the mathematical expression which gives the sum of four rolls of a dice. To do this we could let $x_i = \{1,2,3,4\}$ represent the values of the outcomes of four rolls and then we could write the expression $x_1 + x_2 + x_3 + x_4$ for the sum of four roll.

Then X_i number are called random variables. A random variable is simply an expression whose value is the outcome or a particular experiment. Just as in the case of other types of variable in mathematics, random variables can on different values.

- Let x be a random variable which corresponds to the roll of one dice. We shall assign probabilities to the possible outcomes in this experiment. We do this by assigning to each outcome m_i a nonnegative number $P(mi)$ in such a way that

  $m_i = \{m_1, m_2, m_3, m_4, m_5, m_6\}$ and $P(m_i) = \frac{1}{6}$

  $P(m_1) + P(m_2) + P(m_3) + P(m_4) + P(m_5) + P(m_6) = 1$ Which implies

  $$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

- Let Y be a random variable which represents the number of heads obtained in tossing two coins. The possible outcomes are E= {HH, HT, TH, TT}. $Y_i = \{0,1,2\}$

  $P(y_0) = P(TT) = \frac{1}{4}$: The probability of obtaining 0 head.

  $P(y_1) = P(HT, TH) = \frac{2}{4}$: The probability of obtaining 1 head.

  $P(y_2) = P(HH) = \frac{1}{4}$: The probability of obtaining 2 heads.

  Then $P(y_0) + P(y_1) + P(y_2) = \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$

- Let Z be a random variable which represents the number of kings obtained in a deck of 9 cards. $Z_i = \{0,1,2,3,4\}$, this means that in e deck of cards there are 4 kings. The possible outcomes of drawing them is $\binom{9}{4} = 126$

  $\boldsymbol{P(z_0) = \frac{\binom{4}{0}\binom{5}{4}}{\binom{9}{4}} = \frac{5}{126}}$ **:** The probability of obtaining 0 king.

$$\boldsymbol{P(z_1)} = \frac{\binom{4}{1}\binom{5}{3}}{\binom{9}{4}} = \frac{\mathbf{40}}{\mathbf{126}}$$ **:** The probability of obtaining 1 king.

$$\boldsymbol{P(z_2)} = \frac{\binom{4}{2}\binom{5}{2}}{\binom{9}{4}} = \frac{\mathbf{60}}{\mathbf{126}}$$ **:** The probability of obtaining 2 kings.

$$\boldsymbol{P(z_3)} = \frac{\binom{4}{3}\binom{5}{1}}{\binom{9}{4}} = \frac{\mathbf{20}}{\mathbf{126}}$$ **:** The probability of obtaining 3 kings**.**

$$\boldsymbol{P(z_4)} = \frac{\binom{4}{4}\binom{5}{0}}{\binom{9}{4}} = \frac{\mathbf{1}}{\mathbf{126}}$$ **:** The probability of obtaining 4 kings.

Then $P(z_0) + P(z_1) + P(z_2) + P(z_3) + P(z_4) = \frac{5}{126} + \frac{40}{126} + \frac{60}{126} + \frac{20}{126} + \frac{1}{126} = \frac{126}{126} = 1$

2 DEFINITION AND EXAMPLES

Definition: Let consider a finite set E which contains the elements $x_1, x_2, x_3, \dots, x_n$ are the numbers which are associated to the probability P(x_1), P(x_2), P(x_3),…, P(x_n). The variable X which have the elements or the values on E is said to be a discrete random variable. In other words if the sample space is either finite or countably infinite, the random variable is said to be discrete. Notice that the sample space of the experiment is the set of all possible outcomes and it is denoted by E or Ω.

Example 1: A dice is rolled one time. The set of all possible outcomes is Ω= {1,2,3,4,5,6}

The event A "to have an even number" A={2,4,6} have the probability P(A)=$\frac{3}{6}$

Because P(A_2)=P(A_4)=P(A_6)=$\frac{1}{6}$. Let A be a discrete random variable of having an even number such that A={ A_2, A_4, A_6}

P(A)=P(A_2)+ P(A_4)+ P(A_6)=$\frac{3}{6}$

Example 2: Let toss a coin three times. The set of all possible case is
$\Omega = \{HHH, HTH, HTT, HHT, THH, TTH, THT, TTT\}$ .The card(Ω)=2^3=8 and X_i={0,1,2,3}

Let X be a discrete random variable which corresponds to the number of tails present.

$P(X_0)=P(HHH)=\frac{1}{8}$: no tail appear.

$P(X_1)$ =P(HHT,HTH,THH)=$\frac{3}{8}$: one tail appear.

$P(X_2)$=P(HTT,THT,TTH)=$\frac{3}{8}$: two tails appear.

$P(X_3)$=P(TTT)=$\frac{1}{8}$: three tail appear.

Then P(X)= $P(X_0)$+ $P(X_1)$+ $P(X_2)$+ $P(X_3)$= $\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1$

Probability law : Let x be a discrete random variable which have elements in the set E where their elements are $x_1, x_2, x_3, ..., x_n$ .These elements correspond to the probability $P(x_1)$, $P(x_2)$, $P(x_3)$,…, $P(x_n)$. Hence, the probability law is $\boldsymbol{\sum_{i=1}^{n} P(x_i) = 1.}$

3 DISTRIBUTION FUNCTION

Let X be a discrete random variable, X has the elements which are the set E and these elements are $x_1, x_2, x_3, ..., x_n$. Each element of the set E is associated to the probability

$P(x_1)$, $P(x_2)$, $P(x_3)$,…, $P(x_n)$ such that $\sum_{i=1}^{n} P(x_i) = 1$

Let consider the point {X(Ω) , f[X(Ω)]} by the application or the mapping P_X defined by

$P_X : f[X(\Omega)] \rightarrow [0,1]$: where [0 ,1] is the probability space and f is a probability function.

$$A \rightarrow P_X(A) = P^{-1}[X(A)] \text{ If } X(\Omega)=\{X_i \text{ such that } i \in I\}$$

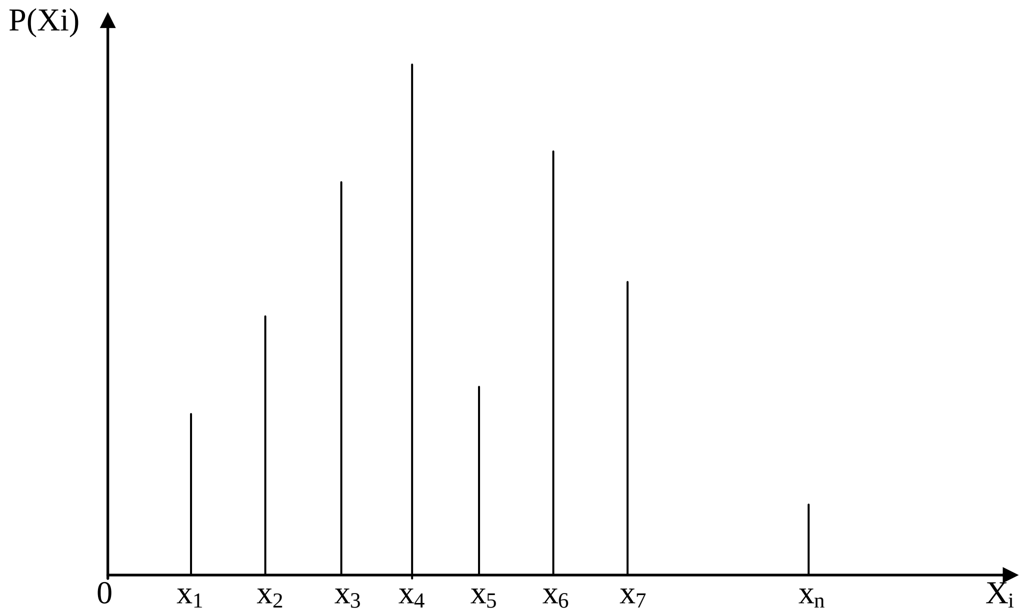
P_X *is the probability* P_i *defined by* **:**

$$\{\textbf{For all } i \in I, \mathbf{P_i = P_x[x_i] = P(X = x_i) = P(x_i)}\}$$

This is what we call the distribution function. The distribution function can also be presented in the following table.

X_i	X_1	X_2	X_3	….	X_i	….	X_n
$P(X_i)$	$P(X_1)$	$P(X_2)$	$P(X_3)$	….	$P(X_i)$	….	$P(X_n)$

Graphic representation: The graphic representation of the distribution function is the histogram or the Bar chart where 0X- axis represent the values of Xi and 0Y-axis represent the values of P(Xi). It is in the following form



Suppose that Height= Xi and Frequency= P(Xi)

Example : Let consider the following distribution table

Xi	1	2	3	4	5	6
P(Xi)	0.1	0.15	x	0.33	0.12	0.09

a. Calculate x
b. Present the graphic of the distribution.

Solution: From $\sum_{i=1}^{n} \mathrm{P}(x_i) = 1$ we can have

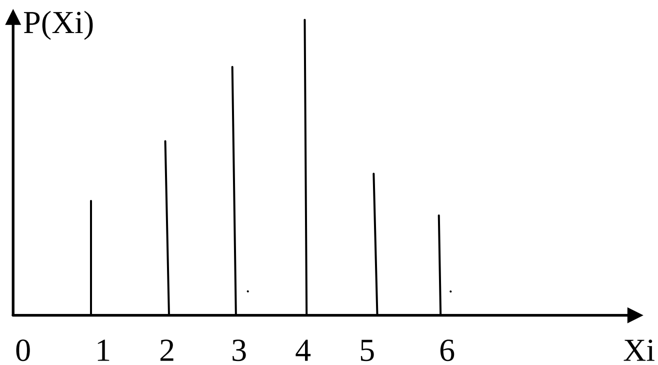
$P(x_1) + P(x_2) + P(x_3) + P(x_4) + P(x_5) + P(x_6) = 1$

Then $0.1+0.15+X+0.33+0.12+0.09=1 \Rightarrow 0.79 + x = 1 \Rightarrow x = 1 - 0.79 \Rightarrow x = 0.21$

Hence the table become

Xi	1	2	3	4	5	6
P(Xi)	0.1	0.15	0.21	0.33	0.12	0.09

Thus the graphic representation become



4 REPARTITION FUNCTION

Definition: The repartition function denoted by F(X) is the function on each value of X correspond F(x) = $\sum_{t=1}^{n} \mathrm{P}(x_t)$ where $\begin{cases} x \leq X \\ F(x) = P(X) \end{cases}$ For x$\in IR$

And we have $\mathbf{F}(\boldsymbol{X_t})= \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \mathbf{P}(\boldsymbol{x_i}) = \boldsymbol{P}(\boldsymbol{X} \leq \boldsymbol{X_t})$

If the sample space is E, then P(E)= $\sum_{i=1}^{n} \mathrm{P}(x_i) = 1$

Remark: In the discrete finite case, we can define F(x) on the following

If X$\leq Xi \Rightarrow F(X) = P(X < x) = 0$,

If Xi$\leq X \leq X_{j+1} \Rightarrow F(X) = P(X < x) = \sum_{i=1}^{j} Pi$ and

If X_n<X $\Rightarrow$ F(x) = P(X<x)=1

Example: Let consider the distribution table of rolling three coins

The se of all possible case is $\Omega = \{HHH, HTH, HTT, HHT, THH, TTH, THT, TTT\}$.

Let X be a discrete random variable such that X_i={0,1,2,3} and

$P(X_0)=\frac{1}{8}$; $P(X_1)= \frac{3}{8}$; $P(X_2)=\frac{3}{8}$; $P(X_3)=\frac{1}{8}$. The distribution function is given by

Xi	0	1	2	3
P(Xi)	1/8	3/8	3/8	1/8

The repartition function is the function F(x) = $\sum_{t=1}^{n} P(x_t)$ obtained by

$x \leq 0$ ⇒ F(x)=0 Or if x$\in (-\infty, 0] \Rightarrow F(X) = 0$

$0 \leq x < 1 \Rightarrow$ F(x)=1/8 Or if x$\in [0\,, 1) \Rightarrow$ F(x)=$\frac{1}{8}$

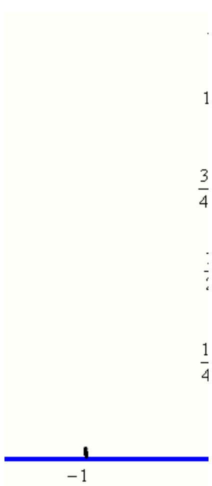
$1 \leq x < 2 \Rightarrow$ F(x)=1/8+3/8=4/8 Or if x$\in [1\,,2) \Rightarrow$ F(x)=$\frac{4}{8}$

$2 \leq x < 3 \Rightarrow$ F(x)=1/8+3/8+3/8=7/8 Or if x$\in [2\,,3) \Rightarrow$ F(x)=$\frac{7}{8}$

3<x ⇒ F(x)=1/8+3/8+3/8+1/8=1 Or if x$\in [3\,, +\infty) \Rightarrow$ F(x)=1

Graphic representation:

The repartition function of the experiment is presented as follow



5 MATHEMATIC EXPECTATION

Definition: The mathematics expectation is the product of the values Xi of the random variable which is associated to the corresponding probability P(Xi). It is written by E(X).

Let X be a discrete random variable where its elements are $x_1, x_2, x_3, \dots, x_n$ and the corresponding probabilities are P(x_1), P(x_2), P(x_3),…, P(x_n) .The mathematic expectation is defined as E(X)=x_1 P(x_1)+x_2 P(x_2)+x_3 P(x_3)+ … +x_n P(x_n)

E(X) = $\boldsymbol{\sum_{i=1}^{n} X_i P(X_i)}$

Example: Let consider the following distribution

Xi	*0*	*1*	*2*	*3*	*4*	*5*
P(Xi)	0.1	0.15	0.25	0.30	0.12	0.08

Calculate E(X)

Solution as E(X) = $\sum_{i=1}^{n} X_i P(X_i)$, Then E(X)=2.43

Xi	**P(Xi)**	**$X_iP(X_i)$**
0 1 2 3 4 5	0.1 0.15 0.25 0.30 0.12 0.08	0 0.15 0.50 0.90 0.48 0.40
		2.43

Properties of E(X): Let X and Y be two discrete random variables and E be the mathematic expectation, The following properties are true

P 1: E(X+Y)= E(X) + E(Y)

P 2: E(X*Y) = E(X)* E(Y)

P 3: E(a*X) = a*E(X) and E(a*Y) = a*E(Y) if a is constant (a $\in IR$)

6 VARIANCE AND STANDARD DEVIATION OF A DISCRETE RANDOM VARIABLE

1. Variance The variance of a discrete random variable is the mean of the square of the range between the value Xi and the mathematic expectation E(X) times the corresponding probability. The variance is written by Var(X) or σ_X^2 and it is defined by

Var(X) $=\boldsymbol{\sum_{i=1}^{n}[Xi - E(X)]^2 P(Xi)}$

after calculation we have **Var(X)** $= \boldsymbol{\sum_{i=1}^{n} Xi^2 P(Xi) - [E(X)]^2}$

This is the most used formula and it is called KOËNING Formula.

From the KOËNING Var(X)= E(X^2) - $[E(X)]^2$ because E(X^2)= $\sum_{i=1}^{n} Xi^2 P(Xi)$

As E(X)=x_1 P(x_1)+x_2 P(x_2)+x_3 P(x_3)+ … +x_n P(x_n)= $\sum_{i=1}^{n} X_i P(X_i)$ then

E(X^2)=$X_1^2 P(X_1) + X_2^2 P(X_2) + X_3^2 P(X_3) + \cdots + X_n^2 P(X_n) = \sum_{i=1}^{n} X_i^2 P(X_i)$ Hence, the formula 3 of variance become

Var(X)= E($\boldsymbol{X^2}$) - $\boldsymbol{[E(X)]^2}$

2. STANDARD DEVIATION: The standard deviation of a discrete random variable X is defined as the square root of the variance of the same random variable. It is written by SD(X) and defined by **SD(X)**=$\boldsymbol{\sqrt{\sum_{i=1}^{n}[Xi - E(X)]^2 P(Xi)}}$

SD(X)=$\boldsymbol{\sqrt{= \sum_{i=1}^{n} Xi^2 P(Xi) - [E(X)]^2}}$ **or**

SD(X)=$\boldsymbol{\sqrt{\mathrm{E}(X^2) - [E(X)]^2}}$

3. Properties: Let X and Y be two random variables

P_1: Var(X+Y)= Var(X) + Var(Y)

P_2: Var(a X) = a^2Var(X) and Var(aY)= a^2 Var(Y) for a=constant

P_3: SD(X+Y)= SD(X)+SD(Y)=$\sqrt{Var(X)}+\sqrt{\text{Var(Y)}}$

P_4: SD(a X)= a^2 SD(X)=a $\sqrt{Var\ (X)}$ and SD(a Y)= a^2 SD(Y)=a$\sqrt{Var\ (Y)}$ For all a=constant.

4. Moment of the order k

The moment of the order K of the finite random variable, is the value m_k where k∈ IN defined by

$$\boldsymbol{m_k = \sum_{i=1}^{n} P(X_i)X_i^k} \textbf{ .}$$

The moment of the order k of the infinite random variable is defined by

$$\boldsymbol{m_k = \sum_{i=1}^{\infty} P(X_i)X_i^k}$$

and this series is absolutely converging.

Example: Let consider the following distribution

Xi	*0*	*1*	*2*	*3*	*4*	*5*
P(Xi)	0.1	0.15	0.25	0.30	0.12	0.08

Calculate E(X), calculate var(x), SD(X) and the moment of the order 4.

Solution

X_i	$P(X_i)$	$X_iP(X_i)$	X_i^2	$X_i^2P(X_i)$	X_i^4	$X_i^4P(Xi)$
0	0.1	0	0	0	0	0
1	0.15	0.15	1	1.5	1	0.15
2	0.25	0.50	4	1	16	4
3	0.30	0.90	9	2.7	81	24.3
4	0.12	0.48	16	1.92	256	30.72
5	0.08	0.40	25	2	625	50

		2.43		9.12		109.17

a. E(X) = $\sum_{i=1}^{n} X_i P(X_i) = 2.43$

b. **Var(X)** =$\boldsymbol{\sum_{i=1}^{n}[Xi - E(X)]^2 P(Xi) = \sum_{i=1}^{n} Xi^2 P(Xi) - [E(X)]^2 = 9.12 - 2.43^2 =}$

Var(x)=$\boldsymbol{9.12 - 5.905}$=3.215

c. SD(X)=$\sqrt{3.215}$

d. $m_4 = \boldsymbol{\sum_{i=1}^{n} P(X_i) X_i^4 = 109.17}$

7 EXAMPLES AND APPLICATIONS

1. Let X be a random variable which is distributed as

Xi	2	4	6
P(Xi)	a	$2a^2$-a	a^2+a-1

a. Determine the value of a

b. Calculate the mathematics expectation, the variance and the standard deviation.

2. Let X be a random variable defined by

Xi	1	2	3	5	7	9	11
P(Xi)	0.05	0.09	0.11	0.15	0.2	0.28	0.22

a. Give the mathematics expectation, the variance and the standard deviation.

b. Give and present graphically the distribution function.

c. Give and present graphically the repartition function.

3. Let X be a random variable defined by

Xi	1	2	3	4	5
P(Xi)	0.25	X	0.18	x	0.37

a. Determine the value of X if the events X=2 and X=4 are equiprobable.

b. Calculate the mathematic expectation, Variance, Standard deviation and the moment of the order 5.

c. Give and present graphically the repartition function.

4. Let X be a discrete random variable such us the probabilities are distributed in the following

Xi	-1	0	2	4
P(Xi)	1/3	1/4	X	1/6

a. Complete the table. Calculate E(X), Var(x) and SD(x).
b. Give and present graphically the distribution function.

5. We throw a coin i/ Three times ii/ Two times iii/ Four times.

Let X be a discrete random variable which correspond to the number of Tails obtained on each draw.

a. Determine the probability law.
b. Calculate E(X), Var(X), SD(X) and m_6(X).
c. Give and present graphically the repartition functions.

6. Let X be a discrete random variable defined by**:** $\begin{cases} \boldsymbol{X(\Omega) = \{1, 2, 3, 4, 5, 6\}} \\ \boldsymbol{P(X = k) = \frac{\omega}{k}} \textbf{ Where } \mathbf{1 \le k \le 6} \end{cases}$

a. Determine ω and calculate E(X), Var(X) and SD(X).

7. A jar contains 4 red marbles and 5 blue marbles. Let X be a discrete random variable which correspond to the drawn red marbles. Determine and verify the probability law. Calculate E(x), Var(X) and SD(x).

8. A jar contains 2n marbles, n white marbles and n black marbles. n marbles are randomly drawn. Let X be the random variable which are associated to the drawn white marbles.

a. Determine the probability law.

b. Calculate E(x), Var(X) and SD(x).

9. Calculate the moment of the order 1, 2, 3, 4, and 5 of the random variable defined by

Xi	1	2	3	4	5	6	7	8

P(Xi)	0.04	0.12	0.14	0.20	0.20	0.14	0.12	0.04

and

Xi	1	2	3	4	5
P(Xi)	0.1	0.2	0.4	0.2	0.1

III 11 CONTINUOUS RANDOM VARIABLES

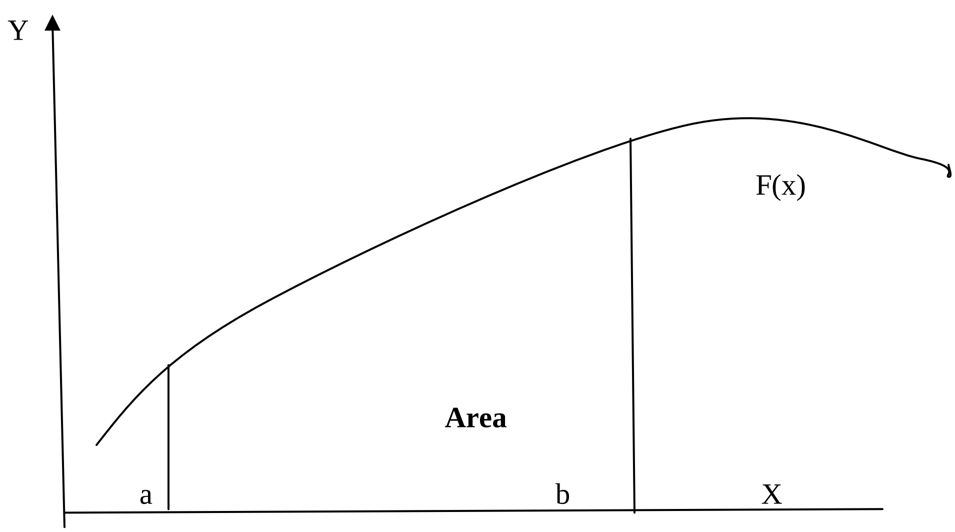
Here are some examples of continuous random variables.

- The mass, in grams, of a bag of sugar packaged by a particular machine.
- The time taken, in minutes to perform a task.
- The height, in centimeter, of a five years girl/boy.
- The life time, in hours of a 100 watt light bulb.

1 DEFINITION AND DENSITY OF PROBABILITY

Let X be a random variable such that the set of the value X(Ω) are the set of points of the interval [a , b].

Let consider the numerical function f defined on IR and continuous toward [a, b].



Such that f : IR→ IR and X → f(x) .The area delimitated by the curve F(X) and the two vertical straight lines X=a and X=b is the probability defined by

$$\boldsymbol{P_r(a \leq x \leq b) = \int_a^b f(x)dx}$$

- In this case we said that X is a continuous random variable, the distribution function f(X) is called **the probability function or the density of probability**. And it is written by $\boldsymbol{P_r(a \le x \le b) = \int_a^b f(x)dx}$

2 MATHEMATIC EXPECTATION

Let X be a continuous random variable of the distribution function $\int_a^b f(x)dx$ in the interval

[a , b]. The mathematic expectation is the value **E(X)=**$\boldsymbol{\int_a^b X\, f(x)dx}$

3 VARIANCE AND STANDARD DEVIATION

Let X be a continuous random variable of the distribution function $\int_a^b f(x)dx$ in the interval

[a , b]. The variance is the value **Var(X) =**$\boldsymbol{\int_a^b X^2\, f(x)dx - [E(X)]^2}$

The standard deviation is the value **SD(X) =** $\boldsymbol{\sqrt{Var(X)} = \sqrt{\int_a^b X^2\, f(x)dx - [E(X)]^2}}$

4 PROBABILITY LAW

Let consider the density of probability F(X) = $\int_a^b f(x)dx$ .The probability law is defined as $\boldsymbol{\int_a^b f(x)dx = 1 \Rightarrow \int_{all} f(x)dx = 1}$

4 REPARTITION FUNCTION

Let X be a continuous random variable. Let f(x) be the numerical function which is continuous toward [a , b] and let $\int_a^b f(x)dx$ be the distribution function . The repartition function is defined as **F(X)=** $\boldsymbol{\int_0^x f(t)dt}\ \ \forall\, \boldsymbol{t} \in [\boldsymbol{a}, \boldsymbol{b}]$

The repartition function verify the following properties $\lim_{x\to a} F(X) = 0\ and\ \lim_{x\to b} F(X) = 1$

5 MOMENT OF THE ORDER K

Let X be a continuous random variable. Let f(x) be the numerical function which is continuous toward [a , b] and let $\int_a^b f(x)dx$ be the distribution function. The moment of the order K is defined as $\boldsymbol{m_k(X) = \int_a^b X^k f(x)dx}$

N.B: If the density of probability is defined on the interval $(-\infty, +\infty)$

- The density of probability is $\boldsymbol{P_r(-\infty \le x \le +\infty) = \int_{-\infty}^{+\infty} f(x)dx}$
- The probability law is $\boldsymbol{\int_{-\infty}^{+\infty} f(x)dx = 1}$

- The mathematic expectation is **E(X)=**$\int_{-\infty}^{+\infty} \boldsymbol{X\, f(x)dx}$
- The variance is **Var(X)** =$\int_{-\infty}^{+\infty} \boldsymbol{X^2\, f(x)dx} - [\boldsymbol{E(X)}]^2$
- The standard deviation **SD(X)** = $\sqrt{\boldsymbol{Var(X)}} = \sqrt{\int_{-\infty}^{+\infty} \boldsymbol{X^2\, f(x)dx} - [\boldsymbol{E(X)}]^2}$
- The repartition function is **F(X)=** $\int_{\mathbf{0}}^{\boldsymbol{x}} \boldsymbol{f(t)dt}$ $\forall\, \boldsymbol{t} \in [\boldsymbol{a}, \boldsymbol{b}]$ and

  $\lim_{x\to-\infty} F(X) = 0 \; and \; \lim_{x\to+\infty} F(X) = 1$
- The moment of the order k is $\boldsymbol{m_k(X)} = \int_{-\infty}^{+\infty} \boldsymbol{X^k f(x)dx}$

II.6 EXAMPLES AND EXERCISES

1. Let X be a continuous random variable which have the density of probability

   $$f(x) = \begin{cases} \frac{1}{2}x & if\ 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$

   a. Calculate $\boldsymbol{P_r(1 \le x \le 1.5)}$

   b. Calculate E(X), Var(X) and SD(X)

   c. Give the repartition function and give the moment of the order 4 and of the order 5.

   d. Give the graphic representation of f(X) and of the repartition function.

   Solution

   $$f(x) = \begin{cases} \frac{1}{2}x & if\ 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$

   a. $\boldsymbol{P_r(1 \le x \le 1.5)} = \int_1^{1.5} f(x)dx = \int_1^{1.5} \frac{1}{2}x\, dx = \left.\frac{x^2}{4}\right)_1^{1.5} = \frac{1}{4}[1.5^2 - 1^2]$=0.3125

   b. **E(X)=**$\int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{X\, f(x)dx} = \int_{\mathbf{0}}^{\mathbf{2}} \boldsymbol{x\ \frac{1}{2}\ x\, dx} = \int_{\mathbf{0}}^{\mathbf{2}} \boldsymbol{x^2 dx} = \left.\frac{x^2}{6}\right)_0^2 = \frac{1}{6}[2^3 - 0^2]$=1.25

   Var(X) =$\int_{\mathbf{0}}^{\mathbf{2}} \boldsymbol{X^2\, f(x)dx} - [\boldsymbol{E(X)}]^2 = \int_{\mathbf{0}}^{\mathbf{2}} \boldsymbol{X^2\ \frac{1}{2}x\, dx} - [\mathbf{1.25}]^2 = \mathbf{0.222}$

   SD(X)=$\sqrt{\boldsymbol{Var(X)}} = \sqrt{\mathbf{0.222}}$**=0.471**

   c. The repartition function is **F(X)=** $\int_{\mathbf{0}}^{\boldsymbol{x}} \boldsymbol{f(t)dt} = \int_{\mathbf{0}}^{\boldsymbol{x}} \boldsymbol{\frac{t}{2}\ dt} = \left.\frac{t^2}{4}\right)_0^x = \frac{X^2}{4}$ and

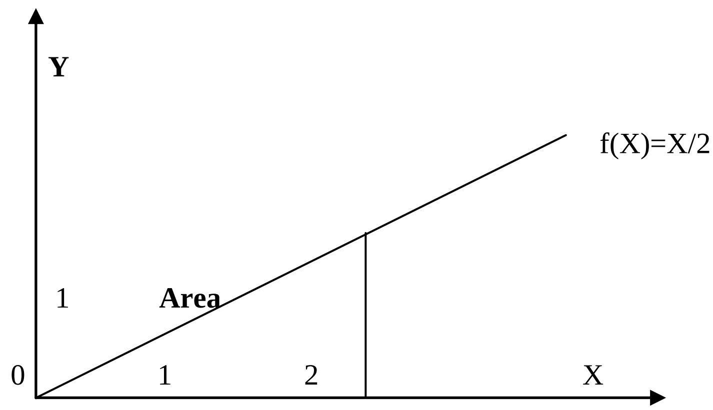
   $$\text{F(X)=}\begin{cases} 0 & if\ x \le 0 \\ \frac{X^2}{4} & if\ 0 \le x \le 2 \\ 1 & if\ x \ge 2 \end{cases}$$

   The moment of the order k is $\boldsymbol{m_k(X)} = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{X^k f(x)dx}$

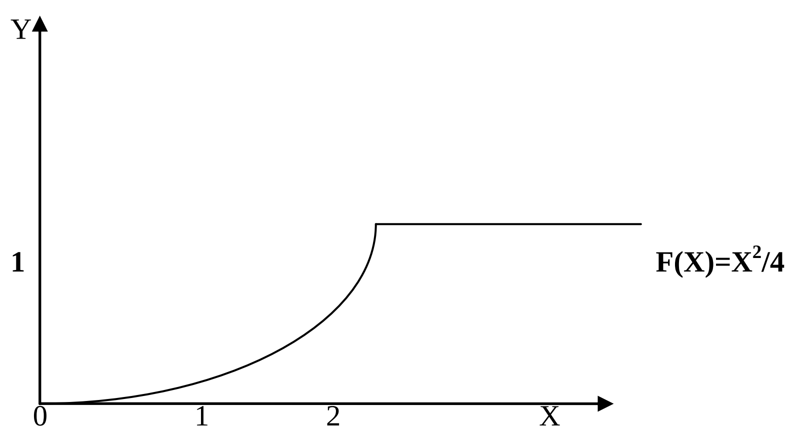
   $$\boldsymbol{m_4(X)} = \int_{\mathbf{0}}^{\mathbf{2}} \boldsymbol{X^4 f(x)dx} = \int_{\mathbf{0}}^{\mathbf{2}} \boldsymbol{X^4 \frac{X}{2} dx} = \left.\frac{x^6}{12}\right)_0^2 = \frac{\mathbf{2^6}}{\mathbf{12}}$$

$$\boldsymbol{m_5(X) = \int_0^2 X^5 f(x)dx = \int_0^2 X^5 \frac{X}{2} dx} = \left.\frac{x^7}{14}\right)_0^2 = \boldsymbol{\frac{2^7}{14}}$$

d. The graphic presentation of f(x)



The graphic representation of the repartition function



2. Let X be a continuous random variable which have the density of probability

$$f(x) = \begin{cases} x & if\ 1 \leq x \leq 4 \\ 0 & otherwise \end{cases}$$

a. Calculate $\boldsymbol{P_r(2 \leq x \leq 3)}$

b. Calculate E(X), Var(X) and SD(X)

c. Give the repartition function and give the moment of the order 3 and of the order 4.

d. Give the graphic representation of f(X) and of the repartition function

3. Let X be a continuous random variable where the repartition function is defined by $\begin{cases} F(X) = 0 & if\ X \leq 0 \\ F(X) = \frac{3x}{4} & if\ 0 < x \leq 1 \\ F(X) = -\frac{x^2}{4} + x & if\ 1 \leq x \leq 2 \\ F(X) = 1 & if\ x > 2 \end{cases}$ Determine and present the density of probability.
4. The continuous random variable X has the Probability Density Function (P.D.F), f(x) where

$$\begin{cases} k(x+2)^2\ if -2 \leq x < 0 \\ 4k \quad if\ 0 \leq x < 1.33 \\ 0 \quad Otherwise \end{cases}$$

   a. Find the value of the constant k and plot f(x)
   b. Calculate P$(-1 \leq x \leq 1)$ and P$(0.5 \leq x \leq 1)$
5. The continuous random variable X has the Probability Density Function (P.D.F), f(x) where

$$f(x)=\begin{cases} \frac{X}{3} & if 0 \leq x \leq 2 \\ -\frac{2X}{3} + 2 & if\ 2 \leq x < 3 \\ 0 & Otherwise \end{cases}$$

   a. Present f(x)
   b. Calculate P(1$\leq x \leq 2.5$)
   c. Find and plot the repartition function.

9. Knowledge, skills, attitudes and values

Understanding and knowledge

- Define a random variable;
- Identify whether a given random variable is discrete or continuous;
- Define the parameters of a discrete random variable;

Skills

- Use the concepts of statistics to compare frequency distribution to probability distribution
- Calculate and interpret the parameters of a random variable(discrete or continuous)
- Construct the probability distribution of a discrete random variable;
- Determine whether a function can serve as probability density function or not;

Values: Appreciate the use of the random variable in the interpretation of statistical data

III.11 Scheme of work

A scheme of work is a series of related learning experience build around one central topic problem or problem area. It is a detailed work plan made in advance for teaching and for learning of the subject content for a given period of time. It forecastes the part of syllabus or of curriculum which will be covered in each lesson and how they would be covered.

The following are reasons for scheming in mathematics

- To make logical ordering of the coverage of topics and subtopic;
- To allocate learning time for each item of the content and assessment;
- To state the depth and scope of treatment of each topic;
- To specify skills and concepts to be learned;
- To outline teaching and learning activities;
- To specify learning and teaching resources;

The following are components of scheme of work

Objective: Must be behavioral even as we embrace constructivism, learning needs to be measured to assist in teachers progress;

Content: Scope of content to be outlined, the syllabus should be consulted at all times as textbooks may not reflect expected level of content scope;

Learning activities or students' activities: based on implementing objectives, it is important for teachers to think of activities that support learning these activities could cognitive, psychomotor, and affective or a combination of all.

Resources and materials: care should be taken when choosing resources, only essential resources must be used.

Evaluation/remarks: feedback for teacher is important for future planning

<u>The following is the format of scheme of work</u>

School…………………………………………………….

Subject/mathematics…………………………………….

Form……………………………………………………..

Year……………………………………………………….

Term……………………………………………………..

Name of teacher………………………………………..

Key unit competence………………………………….

Knowledge & understanding ………………………….

Skills…………………………………………………….

Attitudes & values…………………............................

References……………………………………………..

Week&Date	Lesson	Knowledge/ Understanding	Skills	Attitudes values	Learning/teaching activities	Contents	Observation
1	1						
7^{th}-12^{th} June	2						
	3						
2	1						
	2						
15^{th}-20^{th} June	3						
	4						
3	1						
23^{rd}-28^{th}June	2						
	3						
4	1						
01^{st}-05^{th} July	2						
	3						
	4						
	5						